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ANALYSIS OF MULTIPLE-INPUT STOCHASTIC HYDROLOGIC SYSTEMS

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ABSTRACT

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This report describes the development of a multiple-input stochastic hydrologic system model for the analysis of hydrologic behavior of watersheds. Various components of the hydrologic system are expressed by time series, each containing a trend component, a periodic component, and a stochastic residual component. For modeling the multiple-input system, a Markov-type mathematical formulation is proposed. For illustrative purposes, the model so formulated is applied to the analysis of monthly precipitations and streamflows of the upper Sangamon River basin in Illinois. The results of this study indicate that the proposed model is feasible for the basin and thus can be used for filling missing streamflow data or generating stochastic streamflow sequences.

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PREFACE

This final report relates to the OWRR Project B-038-ILL entitled "Stochastic Analysis of Hydrologic Systems - Phase II," which is a continuation of "Stochastic Analysis of Hydrologic Systems - Phase I" and covers a study period of July 1969 to June 1973. The final report for the Phase I study was published as WRC Research Report No. 26 in December, 1969, and entitled "Stochastic Analysis of Hydrologic Systems" by Ven Te Chow, the Principal Investigator (Publication Board No. 189791, Clearinghouse for Federal, Scientific and Technical Information, now named National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia, 22151).

The main objective of the Phase-II research is to develop general mathematical models for the simulation of stochastic hydrologic processes. The availability of such models is of paramount importance and valuable use in water resources systems planning and development. For this research, the following major achievements have been made:

(1) Completion of the following four major studies:

(a) Presentation of a lumped stochastic hydrologic system model [reported in paper (a) in item (2) below].

(b) Development of a system model for residual stochastic hydrologic processes [reported in paper (h) in item (2) below].

(c) Development of the theory, test, and modeling of stationarity embedded stochastic hydrologic processes [reported in paper (j) in item (2) below and in the Ph.D. thesis by Torelli in item (3) below.

(d) Development of a multiple-input, or spatially distributed stochastic hydrologic system model [reported in this final report].

(2) Publications of the following papers and articles:

(a) Chow, V. T., and Kareliotis, S. J., Analysis of Stochastic Hydrologic Systems, *Water Resources Research*, Vol. 6, No. 6, pages 1569-1582, December 1970.

(b) Chow, V. T., "Stochastic Hydrologic Systems" in Systems Approach to Hydrology," *Proceedings*, U. S. - Japan Bi-Lateral Seminar in Hydrology, Honolulu, Hawaii, January, 1971, Water Resources Publications, Fort Collins, Colorado, pp. 1.1-1.19; discussion, pp. 1.22-1.23, January 1971.

(c) Chow, V. T., "Stochastic Approach in Hydraulics," *Hydraulic Engineering*, Chinese Institute of Hydraulic Engineers, Vol. 13, pp. 199-201, November 1971.

(d) Chow, V. T., "Stochastic Analysis of Hydrologic Systems," *Proceedings*, Fourteenth Congress of the International Association for Hydraulic Research, Paris, France, 29 August to 3 September, 1971, Vol. 5, pp. 265-271, November 1971.

(e) Chow, V. T., "Stochastic Hydraulics-A Challenging Field of Study," in "Stochastic Hydraulics," *Proceedings*, International Symposium on Stochastic Hydraulics, Pittsburgh, Pennsylvania, May 31-June 2, 1971, University of Pittsburgh, pp. 3-8, November 1971.

(f) Chow, V. T., "Hydrologic Modeling - The Seventh John R. Freeman Memorial Lecture," *Proceedings*, Boston Society of Civil Engineers, Vol. 60, No. 5, pp. 1-27, January 1972.

(g) Chow, V. T., and Prasad, T.,"Theory of Stochastic Modeling of Watershed Systems," *Journal of Hydrology*, Vol. 15, No. 4, pp. 261-284, April 1972.

(h) Kareliotis, S. J., and Chow, V. T., "Analysis of Residual Hydrologic Stochastic Processes," *Journal of Hydrology*, Vol. 15, No. 2, pp. 113-130, February 1972.

(i) Kareliotis, S. J., and Chow, V. T.,"Reply to Comments on Analysis of Stochastic Hydrologic Systems," *Water Resources Research*, Vol. 8, No. 1, pp. 163-165, February 1972.

(j) Torelli, L., and Chow, V. T., "Tests of Stationarity of Hydrologic Time Series," *Proceedings*, International Symposium on Uncertainties in Hydrologic and Water Resources Systems, Vol. 1, University of Arizona, Tucson, Arizona, pp. 254-72, December 1972.

(3) Completion of the following theses:

(a) Kareliotis, S. J., "Modeling of a Stochastic Watershed System," Ph.D. thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois, pages xii + 128, 1970.

(b) Torelli, L., "The Analysis of Monthly Hydrologic Time Series," Ph.D. thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois, pages viii + 106, 1973.

(4) Presentation of seven papers at various technical conferences:

(a) "Analysis of Stochastic Hydrologic Systems," by V. T. Chow and S. J. Kareliotis at the 1970 Annual Meeting of the American Geophysical Union in Washington, D.C.

(b) "Nature of Hydrologic Systems," by V. T. Chow at the Second International Seminar for Hydrology Professors in Logan, Utah, on August 2-4, 1970.

(c) "Stochastic Hydrologic Systems," by V. T. Chow at the U.S. - Japan Bi-Lateral Seminar in Hydrology, Honolulu, Hawaii, January 11-15, 1971.

(d) "Stochastic Analysis of Hydrologic Systems," by V. T. Chow at Fourteenth Congress of the International Association for Hydraulic Research, Paris, France, 29 August to 3 September, 1971.

(e) "Stochastic Hydraulics - A Challenging Field of Study," by V. T. Chow at the International Symposium on Stochastic Hydraulics, Pittsburgh, Pennsylvania, on May 31 to June 2, 1971.

(f) "Hydrologic Modeling," by V. T. Chow for the Seventh John R. Freeman Memorial Lecture at the Annual Meeting of the Boston Society of Civil Engineers, Boston, Massachusetts, on 17 February, 1972.

(g) "Tests of Stationarity of Hydrologic Time Series," by L. Torelli and V. T. Chow at International Symposium on Uncertainties in Hydrologic and Water Resources Systems, Tucson, Arizona, December 11-14, 1972.

Many persons participated in this project and contributed to the research. In addition to the Principal Investigator, Ven Te Chow, the following research staff members were involved:

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Most of the results obtained from this Phase-II project have been published elsewhere as listed in item (2) above, but the material presented in this report has not been published before.

I. INTRODUCTION

Modeling of stochastic hydrologic systems in the past has been based on the analysis of the record of a single hydrologic process under consideration, such as the modeling by Thomas and Fiering [1962] for streamflow simulation. Moreover, processes like the streamflow are the result of the interaction of many other processes such as precipitation, infiltration, etc., which vary from watershed to watershed and subsequently affecting the modeling of the streamflow record of the various watersheds. To incorporate these interacting processes in the modeling, the watershed can be modeled as a system with the streamflow as the output and the precipitation as the input. Chow and Ramaseshan [1965] have introduced such an approach for the analysis of floods by considering a single-input, single-output system with a time invariant deterministic system structure. Later in the earlier stage of this research program [Chow, 1969; Chow and Kareliotis, 1970] a modeling technique for a single-input, single-output hydrologic system was developed, taking into account the stochastic behavior of the system's structure. The single-input system is a simplification made on the assumption of a uniform spatial distribution of the precipitation. The present study deals with a multiple-input stochastic hydrologic system and the formulation of a mathematical model for such systems. For illustrative purposes, the model so formulated is applied to the analysis of the upper Sangamon River basin in Illinois. The multiple inputs are the hydrologic data obtained from the various recording precipitation stations over the watershed.

II. FORMULATION OF THE MODEL

Let the records of precipitation inputs be represented by a multiple time series $\{X_{ij}; i = 1,2,...,n; j = 1,2,...,T\}$ where i is the precipitation station number, n is the total number of stations, j is the time periods of the recorded precipitation amount, and T is the total number of time intervals. Similarly, for the single streamflow output the time series is $\{Y_j; j = 1,2,...,T\}$.

It is assumed that the random variable U_t of a time series $\{U_t; t \in T\}$ representing a stochastic hydrologic process can be expressed mathematically by

$$U_t = g_t + p_t + U'_t \tag{1}$$

where g_t is a trend component, p_t is a periodic component, and U'_t is a residual component which remains after the trend and periodic components are removed. The process $\{U'_t; t \in T\}$ has been defined by Kareliotis and Chow [1972] as "the residual hydrologic stochastic process" of the original stochastic hydrologic process $\{U_t; t \in T\}$.

Based on the above assumption and notation, each time series of the precipitation inputs can be expressed as

$$X_{ij} = g_{ij} + p_{ij} + X'_{ij} \tag{2}$$

and of the streamflow as

$$Y_j = g_j + p_j + Y'_j \tag{3}$$

where $\{X'_{ij}; i = 1,2,...,n; j = 1,2,...,T\}$ and $\{Y'_j; j = 1,2,...,T\}$ are the residual hydrologic stochastic processes of the precipitation inputs and the streamflow output, respectively. Before any further analysis, the hydrologic data should be first investigated for possible trends and periodic components, using such tools as power spectra. These components will be then removed, if they exist, and the analysis will proceed with the resulting residual hydrologic stochastic processes.

For the analysis of the multiple-input stochastic hydrologic system, a Markov-type mathematical model of the following form is proposed:

$$Y'_j = \sum_{k=1}^{m} a_k Y'_{j-k} + \sum_{k=0}^{q_1} b_{1k} X'_{1,j-k} + \sum_{k=0}^{q_2} b_{2k} X'_{2,j-k} + \dots + \sum_{k=0}^{q_n} b_{nk} X'_{n,j-k} + \varepsilon_j \qquad (4)$$

where $a_1, a_2, \dots, a_m, b_{10}, b_{11}, \dots, b_{1q_1}, b_{20}, b_{21}, \dots, b_{2q_2}, \dots\dots, b_{n0}, b_{n1}, \dots, b_{nq_n}$ are coefficients to be determined; m is an integer depending on the content of significant autocorrelation in the streamflow record; q_i is also an integer depending on the content of significant crosscorrelation between the streamflow and the precipitation at the i-th station; and ε_j is an independent random residual.

To determine the values of m and q_i, the theory of multiple cross-spectra may be applied, providing a very fast approach for estimating the existence of any significant autocorrelations and crosscorrelations in the data. Then, the coefficients of all the significant terms in Eq. (4) can be estimated by the least-square principle. A brief description of the multiple cross-spectra theory as it is applied in this study will be presented later.

III. ANALYSIS OF THE DATA

A. The Watershed and Hydrologic Data under Study

To demonstrate the feasibility of the proposed multiple-input stochastic hydrologic system model, the watershed chosen as the hydrologic system to be analysed in this study is the upper Sangamon River basin of 550 sq. mi. in size, above Monticello, Illinois, and located in east central Illinois. Figure 1 shows the map of the Sangamon River basin with the locations of the stream gaging station at Monticello and the precipitation gages where hydrologic data were observed for use in the analysis. The lengths of the available data are shown in Table 1.

B. Preliminary Screening of the Data

The preliminary screening of the data involves the investigation of each hydrologic record for its possible trend and periodic components, and the removal of such components, if they exist. The trend components in all records appear to be nonexistent from a careful observation of the data.

The next step involves the search for the periodic components. This consists of a computation of the power spectrum, or the spectrum, of each hydrologic record. Details of the theory of spectrum and its application for the analysis of time series is presented elsewhere [Chow and Kareliotis, 1970]. It must be reminded, however, that the computation of the spectrum involves the computation of the autocovariances C_K for the time series $\{U_t; t \in T\}$ by the formula:

$$C_k = \frac{1}{T-k} \sum_{t=1}^{T-k} U_t U_{t+k} - \frac{1}{(T-k)^2} \sum_{t=1}^{T-k} U_t \sum_{t=1}^{T-k} U_{t+k} \qquad (5)$$

where k is the lag. In the present case, some of the records have intervals

TABLE 1. Available Hydrologic Data for Analysis

Stations	Length of Record
For Precipitation:	
Bloomington	Jan. 1892 - Dec. 1969
Clinton	Jan. 1910 - Dec. 1969
Rantoul	Nov. 1891 - Mar. 1913 Mar. 1940 - Dec. 1969
Roberts	Jan. 1911 - Dec. 1969
Urbana	Sept. 1902 - Dec. 1969
For Streamflow:	
Monticello	Mar. 1908 - Sept. 1912 Nov. 1912 - Dec. 1912 July 1914 - Sept 1969

of missing data. When Eq. (5) is used, the pairs of values of U_t and U_{t+k} for which either one or both values are missing should not be taken into account in the summations. A general computer subroutine has been programmed (see Appendix) which can compute the spectrum of any record with or without intervals of the missing data.

The spectra $\hat{f}(\omega)$ for the six hydrologic records of precipitation are shown in Figs. 2 through 6. These spectra were computed for a maximum lag k = 60. Observing these spectra it can be seen that there are high peaks at the 12-month periodicity and smaller peaks but still higher than other peaks at the 6-month periodicity. Similarly for the spectrum of streamflow as computed and shown in Fig. 7, there is only one peak at the 12-month periodicity. Based on these observations, it is concluded that for precipitation the periodic components consist of 12-month and 6-month periodicities, while for streamflow the periodic component is a 12-month periodicity. Thus, for the general case of the hydrologic time series $\{U_t;\ t \in T\}$ Eq. (1) can be written in the form:

$$U_t = C_1 + C_2 \cos \frac{2\pi t}{12} + C_3 \sin \frac{2\pi t}{12} + C_4 \cos \frac{2\pi t}{6} + C_5 \sin \frac{2\pi t}{6} + U_t' \qquad (6)$$

where C_1 is a coefficient equal to the mean of U_t; C_2, C_3, C_4, and C_5 are the coefficients of the periodic components of 12-month and 6-month periodicities; and U_t' is the residual hydrologic stochastic component with zero mean due to the introduction of the coefficient C_1.

For each hydrologic record the above coefficients are computed using the least-square method. The values of the coefficient are given in Table 2, which also shows the computed variances of the original hydrologic records and the stochastic residual records. A general computer subroutine was written (see Appendix) for the computations which can also handle hydrologic records with intervals of missing data.

TABLE 2. Coefficients of Time Series Models and Variances of Hydrologic Records

Station	C_1	C_2	C_3	C_4	C_5	Variance: Original records	Variance: Stochastic residuals
For Precipitation:							
Bloomington	3.0200	-0.9736	-0.0991	-0.0152	-0.3768	4.0118	3.4767
Clinton	3.2258	-1.0720	-0.2524	-0.0813	-0.4801	4.8724	4.1702
Rantoul	2.9565	-0.9085	-0.0126	0.1821	-0.3502	3.8022	3.3086
Roberts	2.7736	-0.9578	-0.1402	-0.0620	-0.3940	3.3672	2.8348
Urbana	3.0403	-0.8514	-0.0203	-0.0230	-0.3415	3.4456	3.0224
For Streamflow:							
Monticello	0.7982	-0.6151	-0.1968	...	...	1.0539	0.8423

C. Analysis Using Cross-Spectra Theory

A powerful tool in the analysis of the multiple time series of the residual hydrologic stochastic processes is the cross-spectra theory [Goodman, 1965; Bendat and Piersol, 1966; Kareliotis and Chow, 1972]. Suppose that the multiple time series consists of two time series $\{x_t;\ t \in T\}$ and $\{y_t;\ t \in T\}$ and their cross-spectrum is

$$Cr_{xy}(\omega) = p_{xy}(\omega) + iq_{xy}(\omega) \tag{7}$$

where $i = \sqrt{-1}$, ω is the angular frequency, $p_{xy}(\omega)$ is the co-spectrum and $q_{xy}(\omega)$ is the quadrature spectrum. Estimates of co-spectrum and quadrature spectrum are expressed by

$$L[p_{xy}(\omega_j)] = \frac{1}{2\pi} \sum_{k=0}^{m} \frac{1}{2} [C'_{(xy)k} + C'_{(yx)k}] \cos k\omega_j \tag{8}$$

and

$$L[q_{xy}(\omega_j)] = \frac{1}{2\pi} \sum_{k=0}^{m} \frac{1}{2} [C'_{(xy)k} - C'_{(yx)k}] \sin k\omega_j \tag{9}$$

where $\omega_j = \pi j/m$ with $j = 0,1,2,\ldots,m$; and

$$C'_{(xy)0} = C_{(xy)0}$$

$$C'_{(xy)m} = C_{(xy)m}$$

$$C'_{(xy)k} = 2C_{(xy)k} \qquad \text{for } k = 1,2,\ldots,m-1 \tag{10}$$

where $C_{(xy)k}$ is the cross-covariance between $\{x_t\}$ and $\{y_t\}$ of lag k, and m is taken as less than T/10. The estimate of $C_{(xy)k}$ is

$$C_{(xy)k} = \frac{1}{T-k} \sum_{t=1}^{T-k} y_t x_{t+k} - \frac{1}{T-k} \sum_{t=k+1}^{T} x_t \sum_{t=1}^{T-k} y_t \tag{11}$$

The "raw" co-spectrum and quadrature spectrum estimates are "smoothed" by the introduction to Eqs. (8) and (9) of the "Tukey-Hamming" weights given by Blackman and Tukey [1959]:

$$\lambda_k(\omega_j) = 0.54 + 0.46 \cos(\pi k/m) \tag{12}$$

Thus, the final smoothed co-spectrum and quadrature spectrum estimates are

$$U_*(\omega_j) = 0.23\ L_*(\omega_{j-1}) + 0.54\ L_*(\omega_j) + 0.23\ L_*(\omega_{j+1})$$

$$\text{for } j = 1,2,\ldots,m-1$$

$$U_*(\omega_o) = 0.54\ L_*(\omega_o) + 0.46\ L_*(\omega_1)$$

$$U_*(\omega_m) = 0.54\ L_*(\omega_m) + 0.46\ L_*(\omega_{m-1}) \tag{13}$$

where $U_*(\omega)$ and $L_*(\omega)$ are either the smoothed and raw co-spectrum estimates respectively, or the smoothed and raw quadrature spectrum estimates respectively. For the case of a multiple time series $\{X_{1t}, X_{2t}, \ldots, X_{nt}; t \in T\}$, Eq. (7) with the above formulas can be used to compute the cross-spectra matrix:

$$S(\omega) = \begin{vmatrix} Cr_{11}(\omega) & Cr_{12}(\omega) & Cr_{13}(\omega) & \cdots & Cr_{1m}(\omega) \\ Cr_{21}(\omega) & Cr_{22}(\omega) & Cr_{23}(\omega) & \cdots & Cr_{2m}(\omega) \\ Cr_{31}(\omega) & Cr_{32}(\omega) & Cr_{33}(\omega) & \cdots & Cr_{3m}(\omega) \\ \cdots & \cdots & \cdots & Cr_{ij} & \cdots \\ Cr_{n1}(\omega) & Cr_{n2}(\omega) & Cr_{n3}(\omega) & \cdots & Cr_{nm}(\omega) \end{vmatrix} \tag{14}$$

where $Cr_{ij}(\omega)$ are the cross-spectrum between the series $\{X_{it}\}$ and $\{X_{jt}\}$ for $i \neq j$ and the spectrum of the series $\{X_{it}\}$ for $i = j$.

The matrix $S(\omega)$ of Eq. (14) can be written also as

$$S(\omega) = \begin{vmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{vmatrix} \tag{15}$$

where

$$S_{11}(\omega) = \begin{vmatrix} Cr_{11}(\omega) & Cr_{12}(\omega) \\ Cr_{21}(\omega) & Cr_{22}(\omega) \end{vmatrix} \tag{16}$$

$$S_{12}(\omega) = \begin{vmatrix} Cr_{13}(\omega) & \cdots & Cr_{1m}(\omega) \\ Cr_{23}(\omega) & \cdots & Cr_{2m}(\omega) \end{vmatrix} \tag{17}$$

$$S_{21}(\omega) = \begin{vmatrix} Cr_{31}(\omega) & Cr_{32}(\omega) \\ \cdots & \cdots \\ Cr_{n1}(\omega) & Cr_{n2}(\omega) \end{vmatrix} \tag{18}$$

and

$$S_{22}(\omega) = \begin{vmatrix} Cr_{33}(\omega) & \cdots & Cr_{3m}(\omega) \\ \cdots & \cdots & \cdots \\ Cr_{n3}(\omega) & \cdots & Cr_{nm}(\omega) \end{vmatrix} \tag{19}$$

Then a new matrix, called "the partial cross-spectra matrix," can be computed as follows:

$$S_{12k}(\omega) = \begin{vmatrix} Cr_{11k}(\omega) & Cr_{12k}(\omega) \\ Cr_{21k}(\omega) & Cr_{22k}(\omega) \end{vmatrix}$$

$$= S_{11}(\omega) - S_{12}(\omega)\, S_{22}^{-1}(\omega)\, S_{21}(\omega) \tag{20}$$

The $S_{12k}(\omega)$ matrix is a 2 x 2 matrix and its elements are the partial spectra and partial cross-spectra of the series $\{X_{1t}\}$ and $\{X_{2t}\}$ computed by Eq. (20). It must be noted at this point that the computations in Eq. (20) involve the inversion of the square matrix $S_{22}(\omega)$ but such an inversion exists only when the matrix is non-singular, i.e., its determinant is nonzero.

The significance of computing the partial cross-spectra matrix is that it is thereby possible to compute two parameters: the partial coherence and the partial phase, which provide valuable information about the correlation of the series $\{X_{1t}\}$ and $\{X_{2t}\}$ without the influence of the other series of the multiple time series. The partial coherence is given by

$$Ch_{12k}(\omega) = \frac{|Cr_{12k}(\omega)|^2}{Cr_{11k}(\omega)\, Cr_{22k}(\omega)} \tag{21}$$

and the partial phase by

$$\Phi_{12k}(\omega) = \tan^{-1} \frac{\text{imaginary part of } Cr_{12k}(\omega)}{\text{real part of } Cr_{12k}(\omega)} \tag{22}$$

The partial coherence is a measure of the correlation of the two time series without the influence of the other time series. When plotted against ω, it produces the coherence diagram. Similarly, the partial phase will produce the phase diagram which is a measure of the phase relationship between the two series. [Granger, 1964].

In the present study the multiple time series consists of the time series of the residual hydrologic stochastic processes of the output streamflow and the input precipitations, i.e., $m = 6$. To study the dependence of the residual hydrologic stochastic process of streamflow with each one of the residual hydrologic stochastic processes of the precipitation, the partial cross-spectra of streamflow and each one of the precipitation records are computed. Thus, in the multiple time series, $\{X_{1t}\} = \{Y_1\}$ always represents the record of streamflow and $\{X_{2t}\}$ is the precipitation record which is under investigation for determining its influence on the streamflow. For example, if the precipitation record at Bloomington is investigated, the partial cross-spectra matrix of the streamflow and the Bloomington precipitation record has to be computed. Then $\{X_{1t}\}$ will be the streamflow record, $\{X_{2t}\}$ be the Bloomington precipitation record and $\{X_{3t}\}$, $\{X_{4t}\}$, $\{X_{5t}\}$, $\{X_{6t}\}$ be the rest of the precipitation records. A general computer program was prepared (see Appendix) for determining the partial cross-spectra matrix and then the partial coherence and the partial phase according to the formulas mentioned previously, for any number of precipitation stations and any combination of missing data from the records. However, this attempt to compute the partial cross-spectra matrix was unsuccessful because, as it was noticed before, the computation involves the inversion of matrix $S_{22}(\omega)$ of Eq. 19 which happens to be singular; i.e., its determinant is equal or very close to zero. The cause of singularity in matrix $S_{22}(\omega)$ is believed to be attributed to the high correlated precipitation records. It can be proved very easily that if the time series $\{X_{3t}\}$, $\{X_{4t}\}$, $\{X_{5t}\}$, and $\{X_{6t}\}$ are highly correlated then each pair of columns of matrix $S_{22}(\omega)$ have elements which are proportional to each other and as a result of this the matrix is singular. The high degree of correlation between the precipitation records can be shown to be true by computing the coherence diagrams for a number of pairs of precipitation stations. Four such typical coherence diagrams are plotted in Figs. 8, 9, 10, and 11, which indicate a high degree of correlation among the precipitation stations. As a result of this analysis it is concluded that the precipitation record at one station only is sufficient for the description of the precipitation influence on streamflow.

The analysis here indicates that the record at one station is sufficient to describe the dependence of streamflow on precipitation. Thus, Eq. (4) can be written in the following form:

$$Y'_j = \sum_{k=1}^{m} a_k \, Y'_{j-k} + \sum_{k=0}^{q} b_k \, X'_{i,j-k} + \varepsilon_j \tag{23}$$

where $\{X'_{it}\}$ can be any of the five residual precipitation records. Looking at Fig. 7 of the spectrum of streamflow, it can be concluded that the content of significant autocorrelation in streamflows is equal to one [Kendall and Stuart, 1966] and therefore, Eq. (23) can be further simplified as follows:

$$Y'_j = a \, Y'_{j-1} + \sum_{k=0}^{q} b_k \, X'_{i,j-k} + \varepsilon_j \tag{24}$$

To estimate the magnitude of q, the phase diagram is computed between the streamflow and one record of the precipitation stations. Figure 12 shows the phase diagram between the streamflow and the precipitation at Urbana. Since this diagram does not indicate any trend [Granger, 1964], it can be concluded that q = 0. Thus, Eq. (24) can be written as

$$Y'_j = a\,Y'_{j-1} + b\,X'_{ij} + \varepsilon_j \tag{25}$$

where $\{X'_{ij}\}$ is any of the five residual precipitation records. The coefficients a and b in Eq. (25) are computed by the least-square method. In the first trial of computations, the record $\{X'_{ij}\}$ is that at Bloomington and the results indicate that the variance of ε_j is equal to 0.4816 while the original variance of Y'_j was 0.8423 (Table 2); i.e., a reduction in the variance of about 43%. Similarly, for the record at Clinton, the variance is 0.4582 or a 46% reduction; for the record at Roberts, it is 0.5075 or a 40% reduction; and for the record at Urbana, it is 0.4473 or a 47% reduction. The record at Rantoul was not tried because of the too large amount of missing data. As a result of these computations, the precipitation record at Urbana which provides the greatest reduction in variance is recommended for use in Eq. (25). The coefficients for the Urbana record are: a = 0.3538 and b = 0.2897.

D. Analysis Using Principal Components Theory

After having concluded that one variable from the five precipitation variables is sufficient for the description of the streamflow dependence on the precipitation, the question on which precipitation station record should be used in a regression equation for the streamflow is then considered. For this question, a principal component transformation of the five precipitation variables is made. That component which indicates the greatest amount of the streamflow variation is then selected.

The principal component transformation is a mathematical technique [Kendall and Stuart, 1966] which is applicable to multiple time series. According to this technique, the original multiple time series is transformed to a new multiple time series where the time series are no longer cross-correlated as in the original multiple time series. Suppose that the multiple time series of the precipitation inputs is $\{X_{1t}, X_{2t}, \ldots, X_{nt}; t \in T\}$ [noting the difference from the corresponding $\{X_{2t}, X_{3t}, \ldots, X_{n+1,T}\}$ defined previously for computer programs], or in a matrix form:

$$X = \begin{vmatrix} X_{11} & X_{12} & \cdots & X_{1T} \\ X_{21} & X_{22} & \cdots & X_{2T} \\ \cdots & \cdots & \cdots & \cdots \\ X_{n1} & X_{n2} & \cdots & X_{nT} \end{vmatrix} \tag{26}$$

Then, the transformation involves the computation of a new multiple time series:

$$\zeta = \begin{vmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1T} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2T} \\ \cdots & \cdots & \cdots & \cdots \\ \zeta_{n1} & \zeta_{n2} & \cdots & \zeta_{nT} \end{vmatrix} \tag{27}$$

where the time series $\{\zeta_{1t}; t \in T\}$, $\{\zeta_{2t}; t \in T\}$, ..., $\{\zeta_{nt}; t \in T\}$ are not cross-correlated and they are called "principal components." It must be noted, however, that by cross-correlation in this analysis it is meant cross-correlation of zero lag and the principal component analysis eliminates only the zero lag cross-correlation. But, looking at Fig. 13, which is a typical phase diagram for the precipitation data, it is obvious [Granger, 1964] that the precipitation data are only cross-correlated at zero lag. Thus, the principal component transformation of the precipitation data will provide a set of time series which are not cross-correlated at any lag.

The computation of matrix ζ involves the computation of the following matrix:

$$A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{3m} \end{vmatrix} \qquad (28)$$

which is related to matrix X by the following equation:

$$\zeta = AX \qquad (29)$$

If C is the matrix of covariances and cross-covariances of lag zero of the data in matrix X; i.e.,

$$C = \begin{vmatrix} C_{11} & C_{12} & \cdots & C_{1m} \\ C_{21} & C_{22} & \cdots & C_{2m} \\ \cdots & \cdots & C_{ij} & \cdots \\ C_{n1} & C_{n2} & \cdots & C_{nm} \end{vmatrix} \qquad (30)$$

where C_{ij} is computed according to Eq. (11) for k = 0. Then it can be proved [Kendall and Stuart, 1966] that the rows of matrix A are the latent vectors of Matrix C and the latent rods of Matrix C are the covariances of zero lag or the variances of the time series of the multiple time series $\{\zeta_{1t}, \zeta_{2t}, \ldots, \zeta_{nt}, t \in T\}$.

TABLE 3. Matrix C and its Latent Roots and Latent Vectors

Matrix C (dispersion matrix)				Latent roots (eigenvalues)	Latent vectors (eigenvectors)			
3.47987	2.88683	2.45393	2.41597	10.95903	0.51474	0.56335	0.44055	0.47287
2.88683	4.18009	2.33947	2.75407	1.15100	0.37937	-0.69895	0.59180	-0.31163
2.45393	2.33947	2.81457	2.12946	0.80936	-0.48390	-0.32114	0.11031	0.80656
2.41597	2.75407	2.12946	3.06413	0.61928	-0.59747	0.30163	0.66598	-0.32944

In the present analysis only data from four precipitation stations were used because the data from Rantoul were ignored due to the large amount of missing parts. Table 3 shows the computed matrix C latent rods and vectors. In the literature, it is often to call the latent rods as eigenvalues, the latent vectors as eigenvectors, and Matrix C as dispersion matrix. In this computation the series $\{X_{1t}\}$ is Bloomington, $\{X_{2t}\}$ for Clinton, $\{X_{3t}\}$ for Roberts and $\{X_{4t}\}$ for Urbana. Then using the computed values for matrix A (Table 3) the new series $\{\zeta_{1t}\}$, $\{\zeta_{2t}\}$, $\{\zeta_{3t}\}$, $\{\zeta_{4t}\}$ were computed.

In order to determine which of the new time series will explain the greatest variation of the streamflow record, the coherence diagrams for each of the time series $\{\zeta_{it}\}$ and for the residual hydrologic stochastic process of the streamflow were computed. These diagrams are shown in Figs. 14, 15, 16, and 17. It is obvious that the time series $\{\zeta_{1t}\}$ is the best for the purpose due to the high coherence. To determine the lag of the cross-correlation, the phase diagram between the time series $\{\zeta_{1t}\}$ and $\{Y_t\}$ is plotted in Fig. 18. This graph indicates a zero lag in the crosscorrelation [Granger, 1964].

Going back to Fig. 7 of the spectrum of the streamflow, it can be observed that the streamflow is also serially correlated with lag one due to the shape of the spectrum [Kendall and Stuart, 1966]. Thus, the streamflow residuals will be serially correlated with lag one and cross-correlated to $\{\zeta_{1t}\}$ with lag zero. Based on this conclusion, the following model can be suggested:

$$Y'_j = a'Y'_{j-1} + b'\,\zeta_{1j} + \varepsilon_j \qquad (27)$$

where the coefficients a' and b' are computed by the least-square method as $a' = 0.3488$ and $b' = 0.1611$.

The variance of ε_j is found to be 0.4184; i.e., there is a 50% reduction of the original variance of 0.8423. Therefore, the model of Eq. (27) is not any significantly better than the model of Eq. (25) because the reduction in variance is almost the same in both models. Since the modeling procedure for deriving Eq. (25) involves less computations than the one for Eq. (27), the former is recommended for the modeling of the single-input, single-output watershed systems.

IV. CONCLUSIONS AND COMMENTS

As a result of the analysis of the hydrologic data from the upper Sangamon River basin for the proposed modeling of multiple-input stochastic hydrologic systems, the following conclusions and comments may be given:

(a) The hydrologic watershed system can be theoretically simulated as a multiple-input, single-output system and orderly analyzed by the multiple cross-spectra theory.

(b) For the monthly data from the upper Sangamon River basin the single-input, single-output system is the appropriate model due to the highly correlated precipitation inputs.

(c) Principal component transformation of the precipitation data does not provide a component which would be better cross-correlated with the streamflow than any of the original time series of the precipitation inputs.

(d) The monthly precipitation records consist of 12-month and 6-month periodicities while the streamflow record consists only of a 12-month periodicity. The effect of the watershed storage may be the cause for the filtering of the 6-month periodicity.

(e) The residual hydrologic stochastic process of streamflow is serially correlated due to the storage effect of the watershed and it is also cross-correlated to the residual hydrologic stochastic process of precipitations.

(f) The proposed model can be used for filling in missing streamflow data or generating stochastic streamflow sequences.

(g) The proposed approach of the multiple-input, single-output system can be further checked using data of shorter time intervals than one month. For example, weekly data must be more sensitive to the spatial distribution of precipitation than the monthly data used in the present study.

(h) Finally, the method recommended herein can be improved by incorporating additional system components such as evapotranspiration, watershed storage, etc.

V. ACKNOWLEDGMENTS

This report is a result of the research project on "Stochastic Analysis of Hydrologic Systems - Phase II" which began in July 1969 and was completed in June 1973. The work upon which this publication is based was supported in part by funds provided by the United States Department of the Interior as authorized under the Water Resources Act of 1964, Public Law 88-379, Agreement No. 14-31-0001-3076. Under the direction of the Project Investigator, the hydrologic data used in this study were mainly collected by Dr. Gonzalo Cortes-Rivera and the mathematical analysis and computer programming were largely performed by Dr. Sotirios J. Kareliotis.

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VII. FIGURES

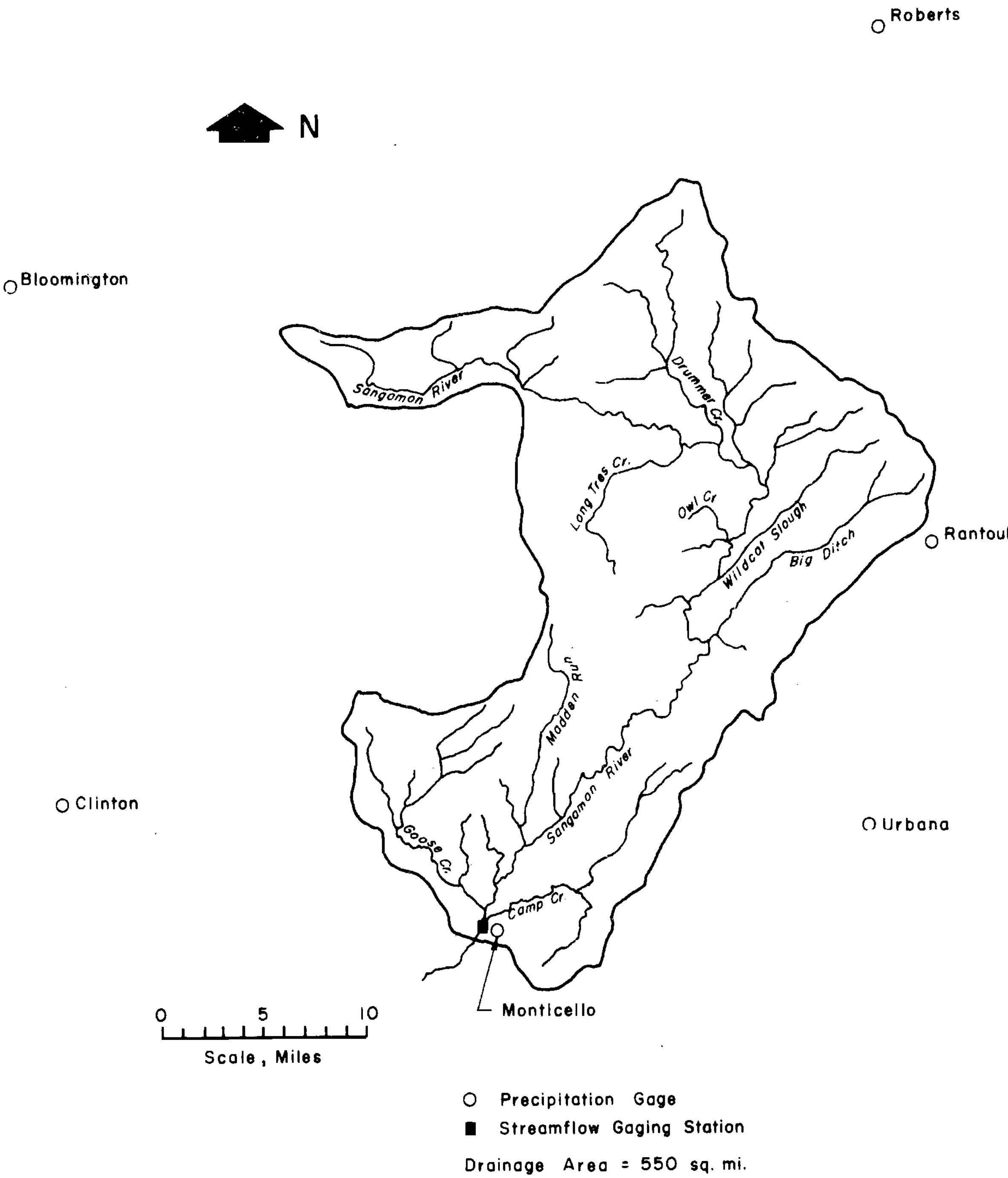

FIG. 1. SANGAMON RIVER BASIN ABOVE MONTICELLO, ILLINOIS

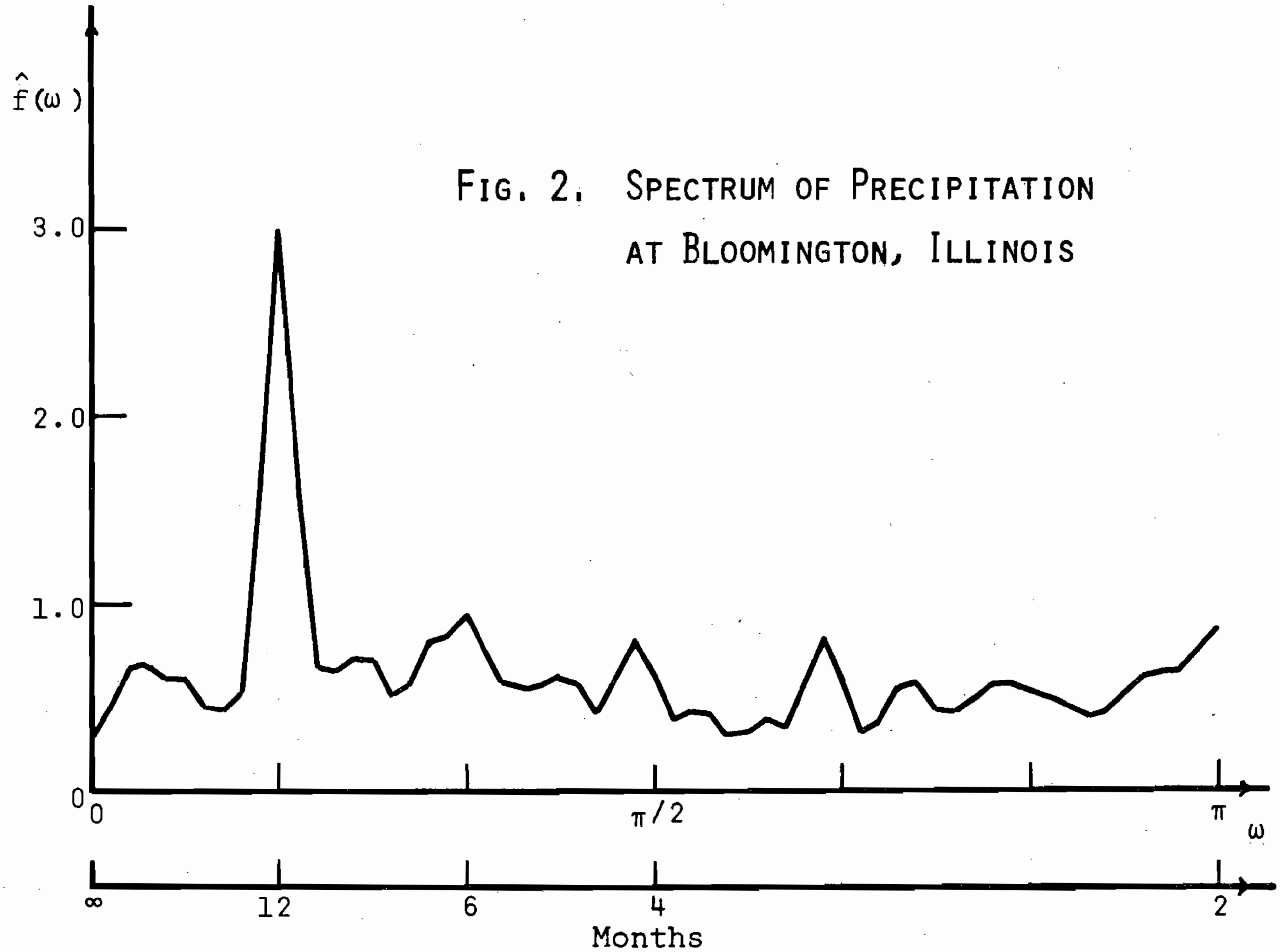

FIG. 2. SPECTRUM OF PRECIPITATION AT BLOOMINGTON, ILLINOIS

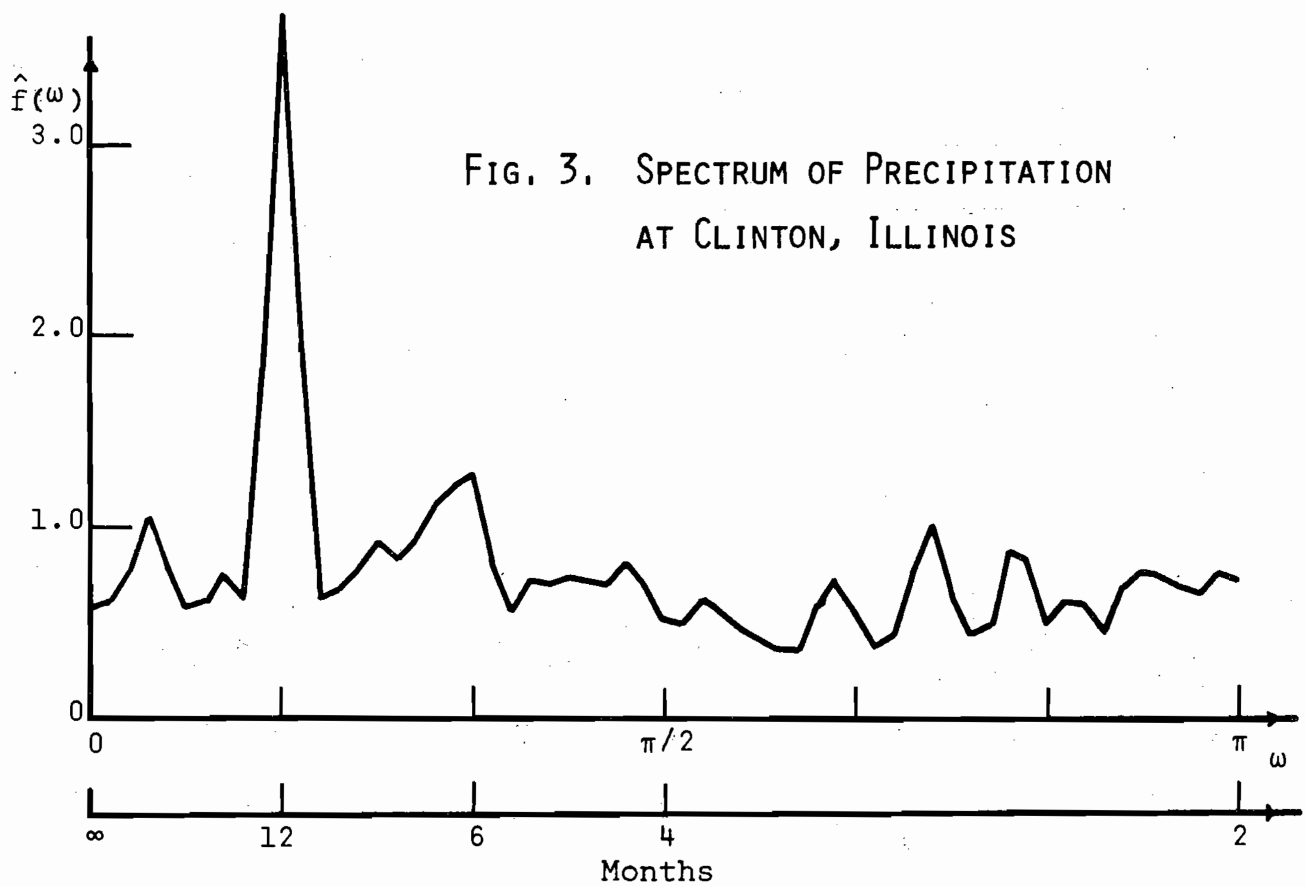

FIG. 3. SPECTRUM OF PRECIPITATION AT CLINTON, ILLINOIS

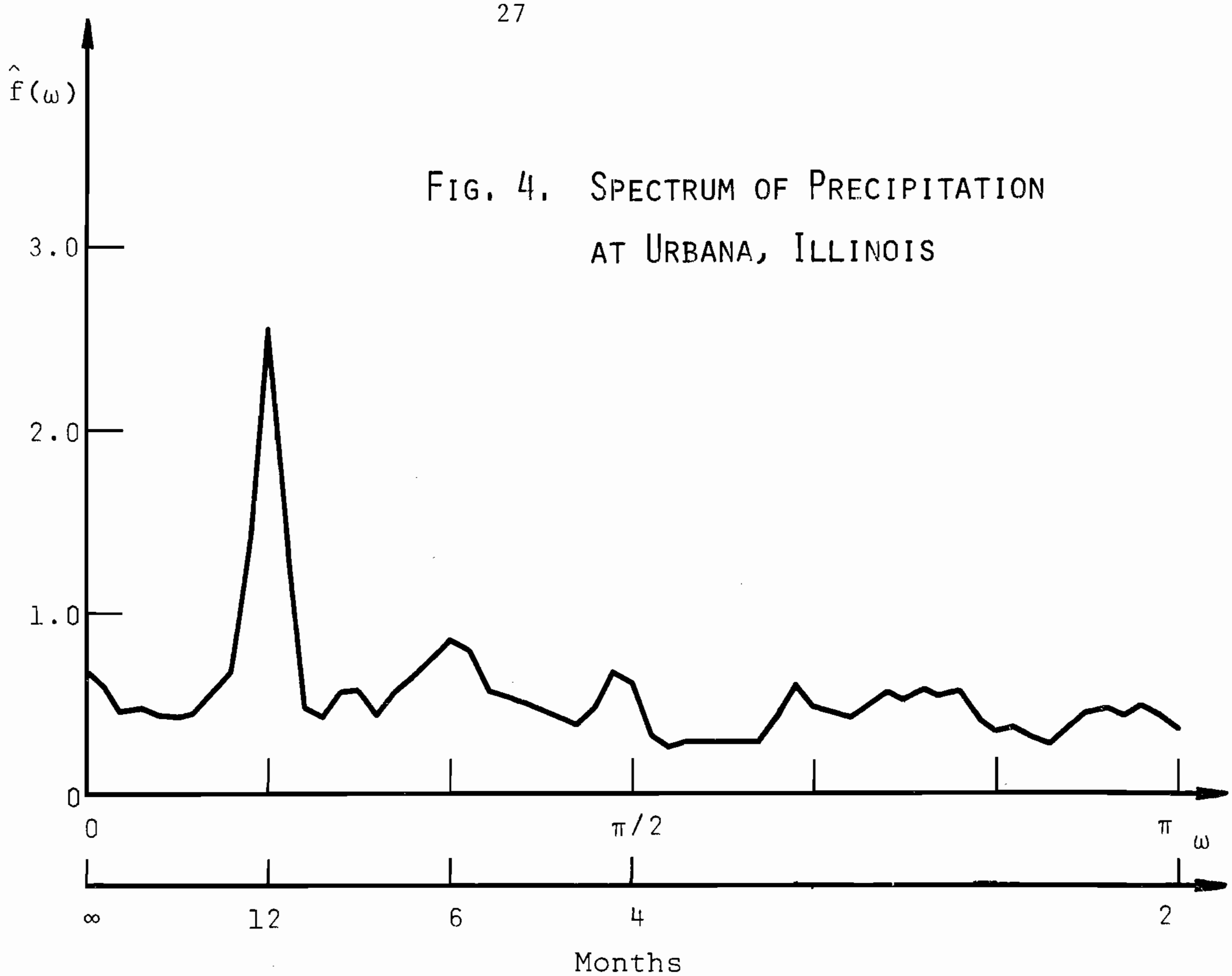

Fig. 4. Spectrum of Precipitation at Urbana, Illinois

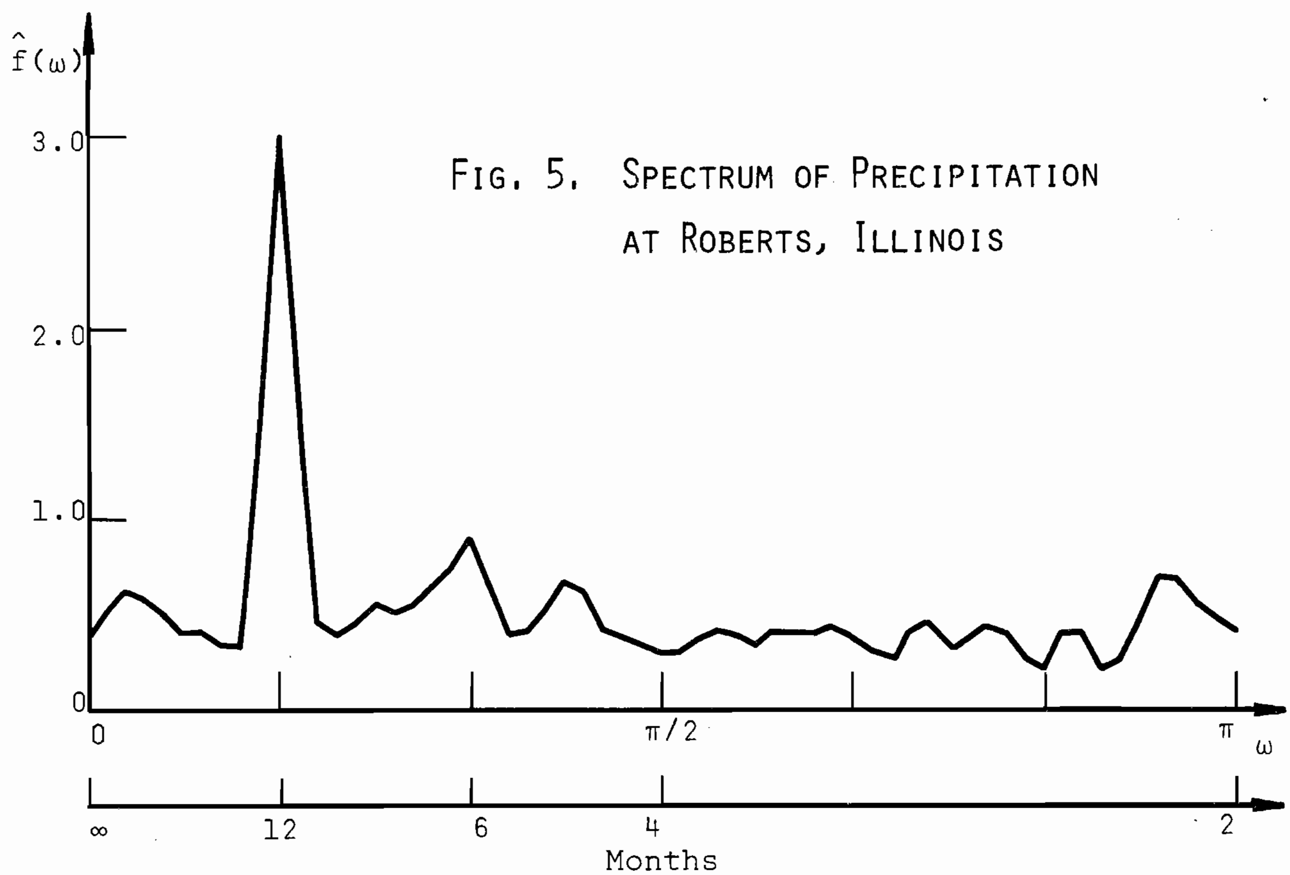

Fig. 5. Spectrum of Precipitation at Roberts, Illinois

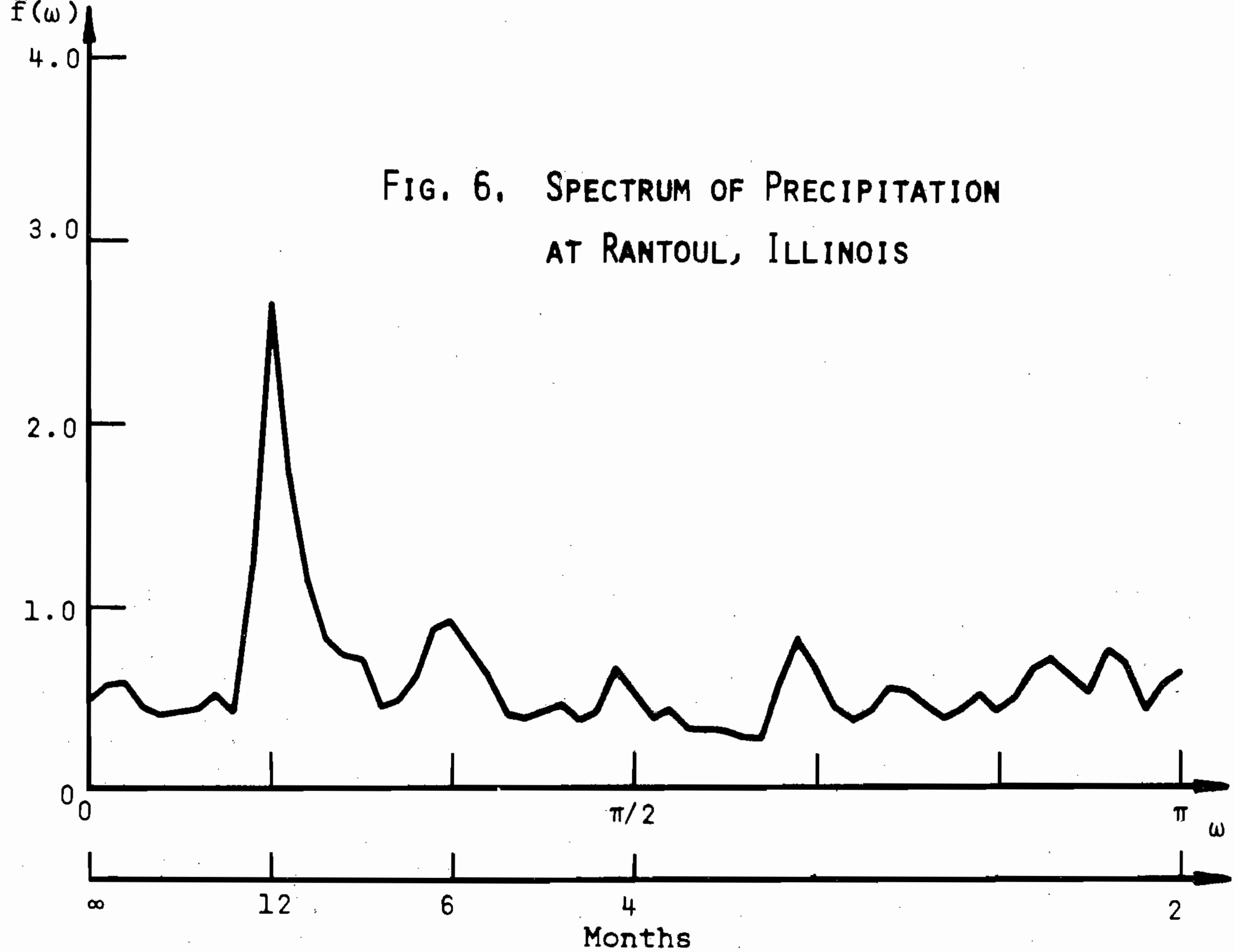

FIG. 6. SPECTRUM OF PRECIPITATION AT RANTOUL, ILLINOIS

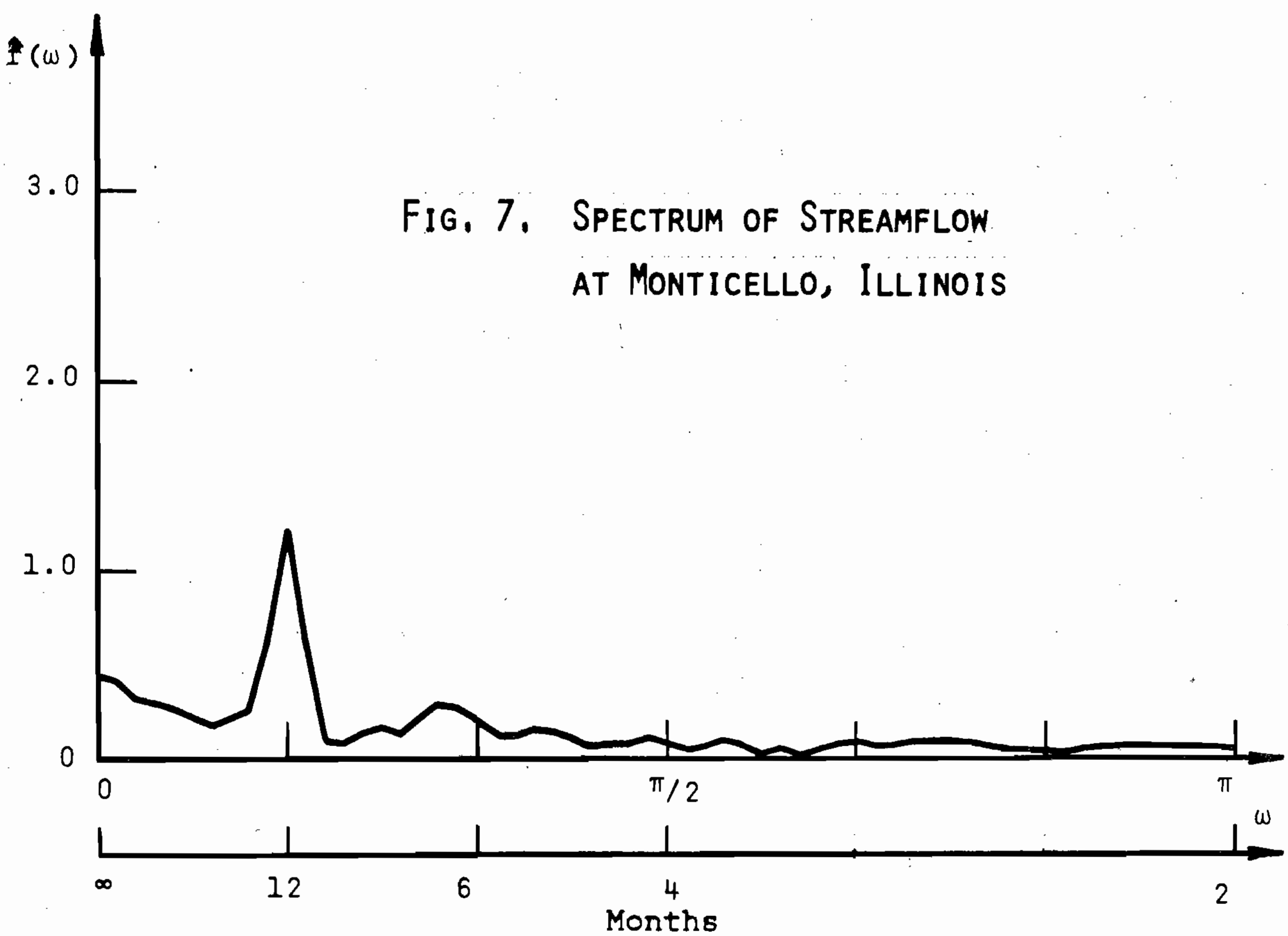

FIG. 7. SPECTRUM OF STREAMFLOW AT MONTICELLO, ILLINOIS

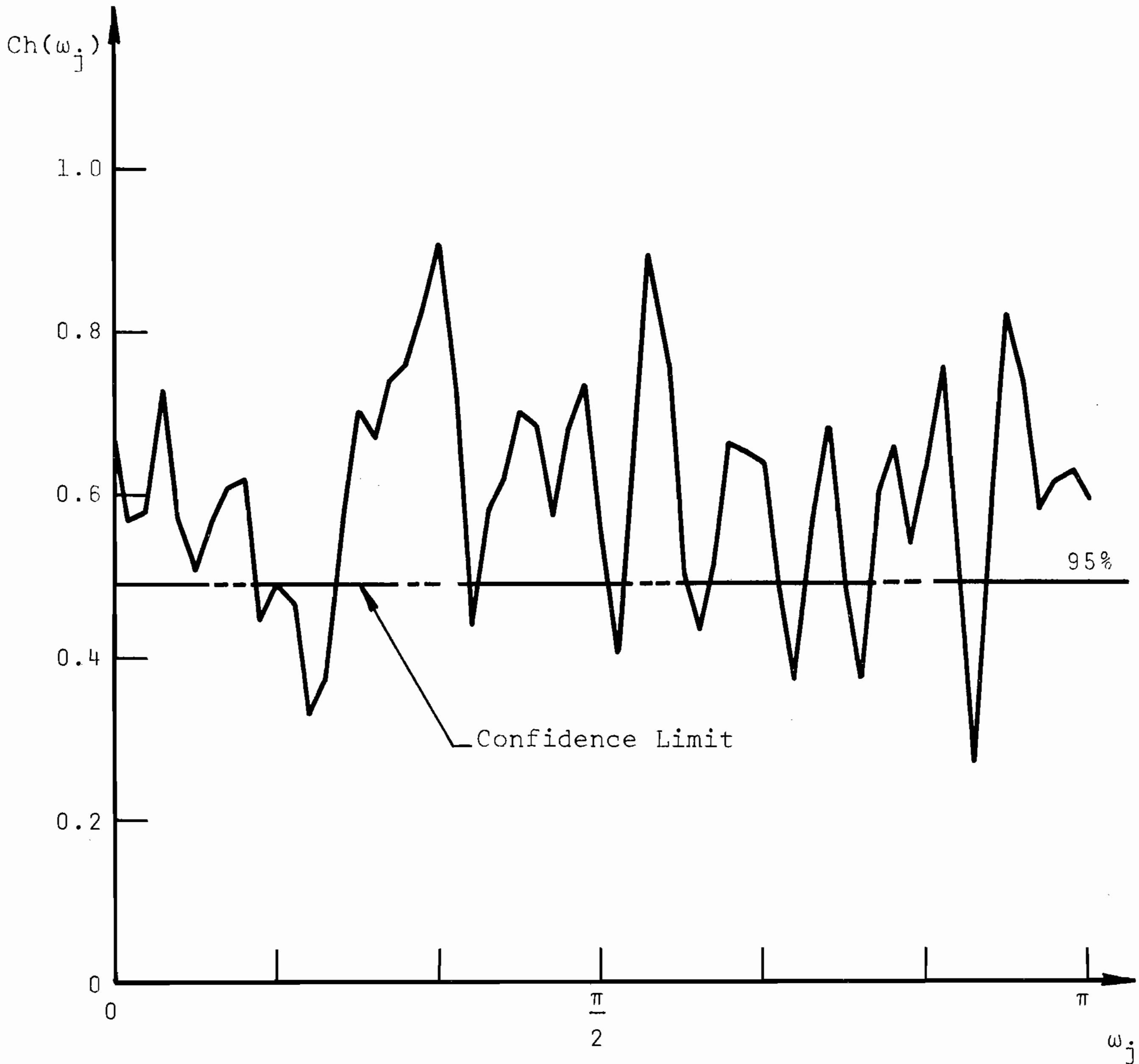

FIG. 8. PARTIAL COHERENCE OF THE PRECIPITATION STOCHASTIC RESIDUALS AT BLOOMINGTON AND CLINTON, ILLINOIS

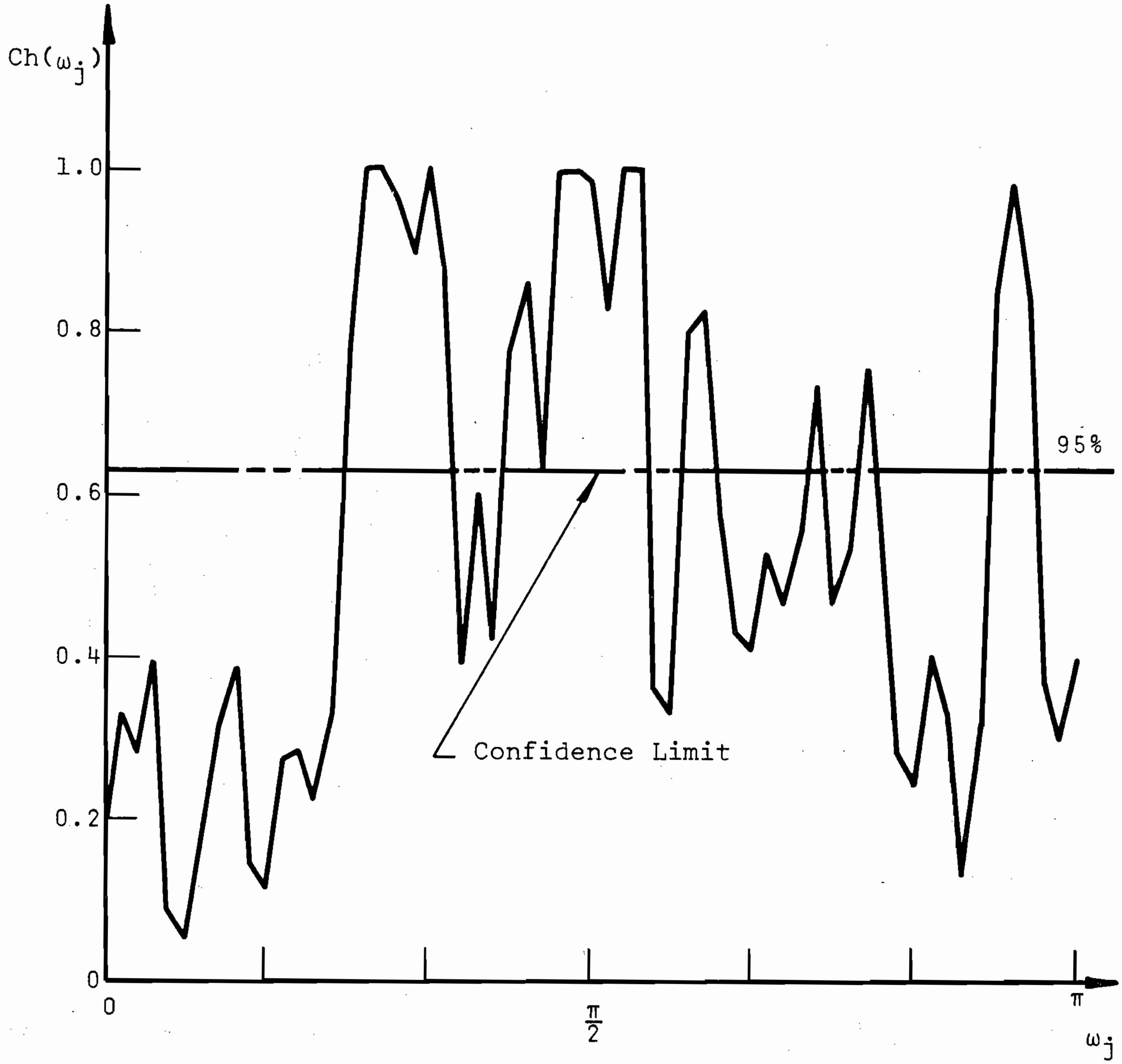

FIG. 9. PARTIAL COHERENCE OF THE PRECIPITATION STOCHASTIC RESIDUALS AT CLINTON AND RANTOUL, ILLINOIS

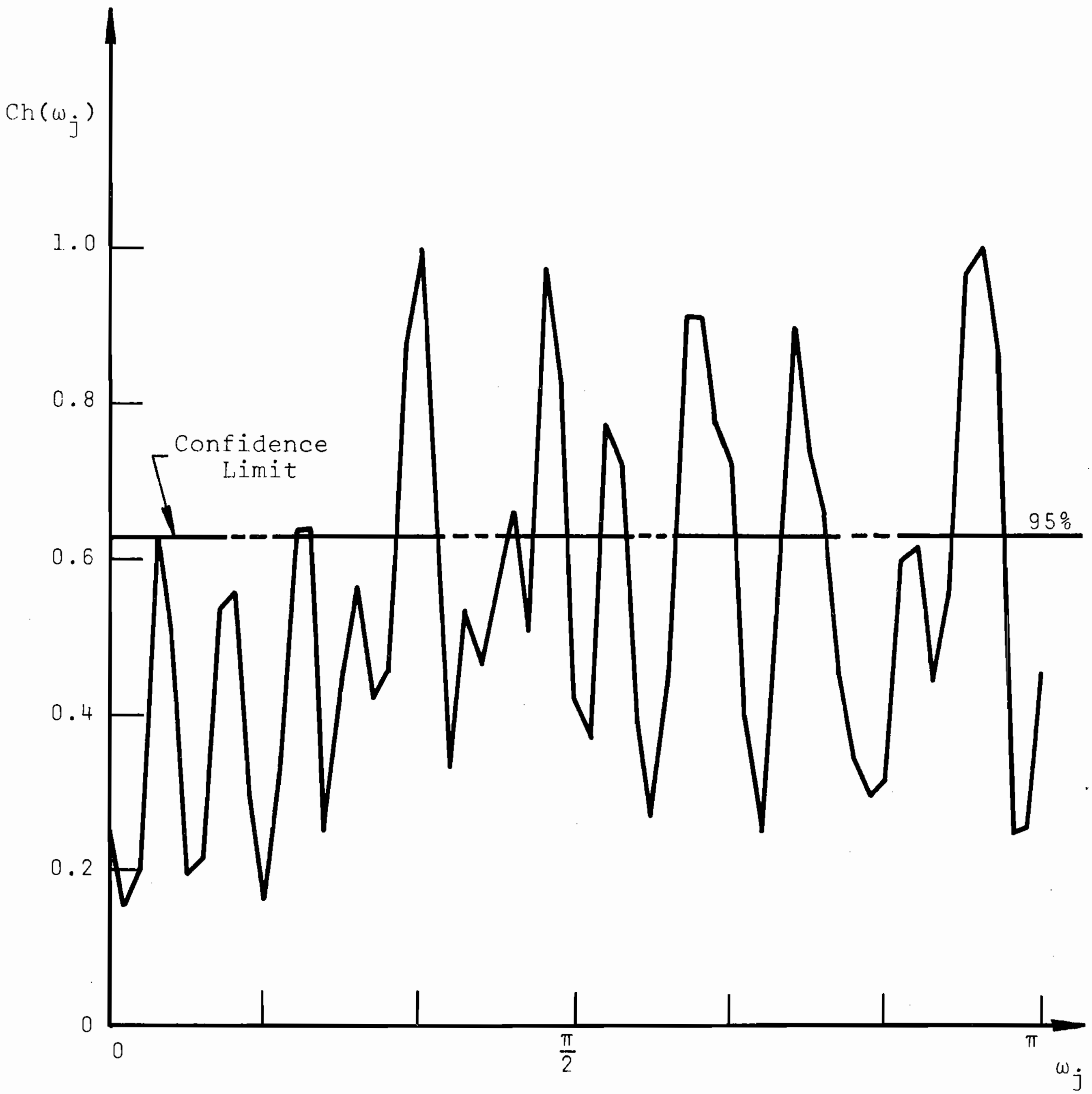

FIG. 10. PARTIAL COHERENCE OF THE PRECIPITATION STOCHASTIC RESIDUALS AT RANTOUL AND ROBERTS, ILLINOIS

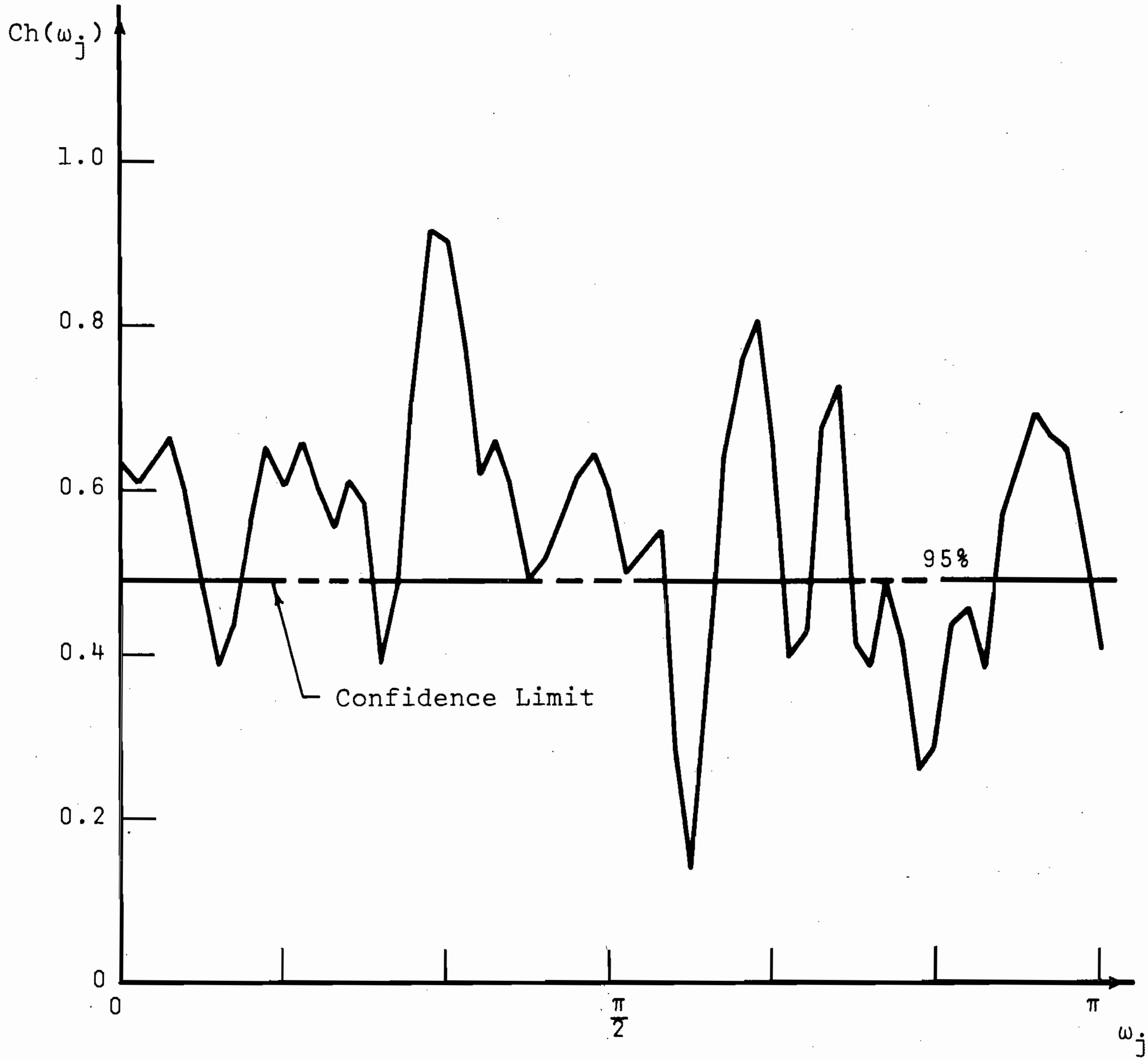

FIG. 11. PARTIAL COHERENCE OF THE PRECIPITATION STOCHASTIC RESIDUALS OF ROBERTS AND URBANA, ILLINOIS

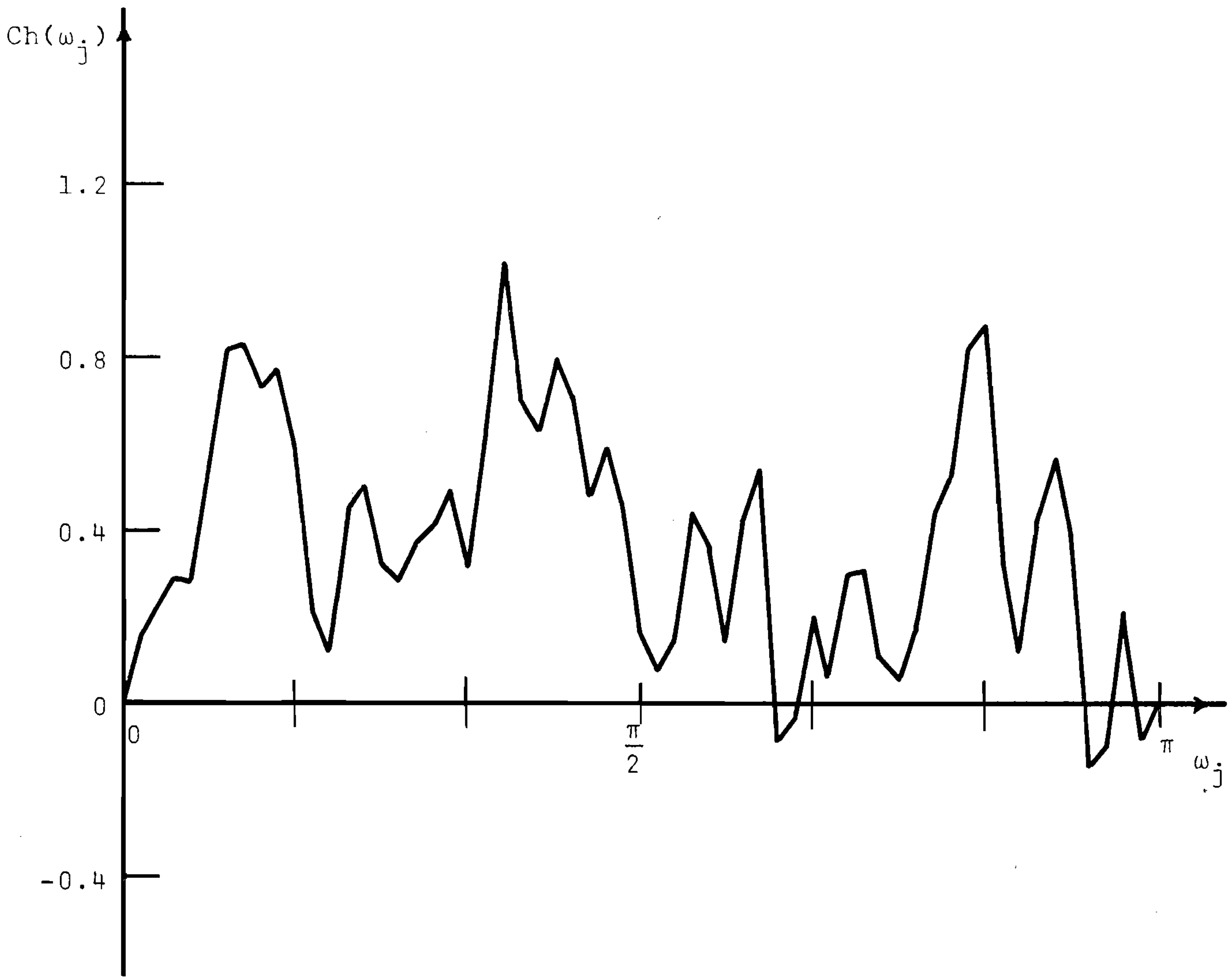

FIG. 12. PHASE DIAGRAM BETWEEN THE STOCHASTIC RESIDUALS OF STREAMFLOW AT MONTICELLO, ILLINOIS AND OF PRECIPITATION AT URBANA, ILLINOIS

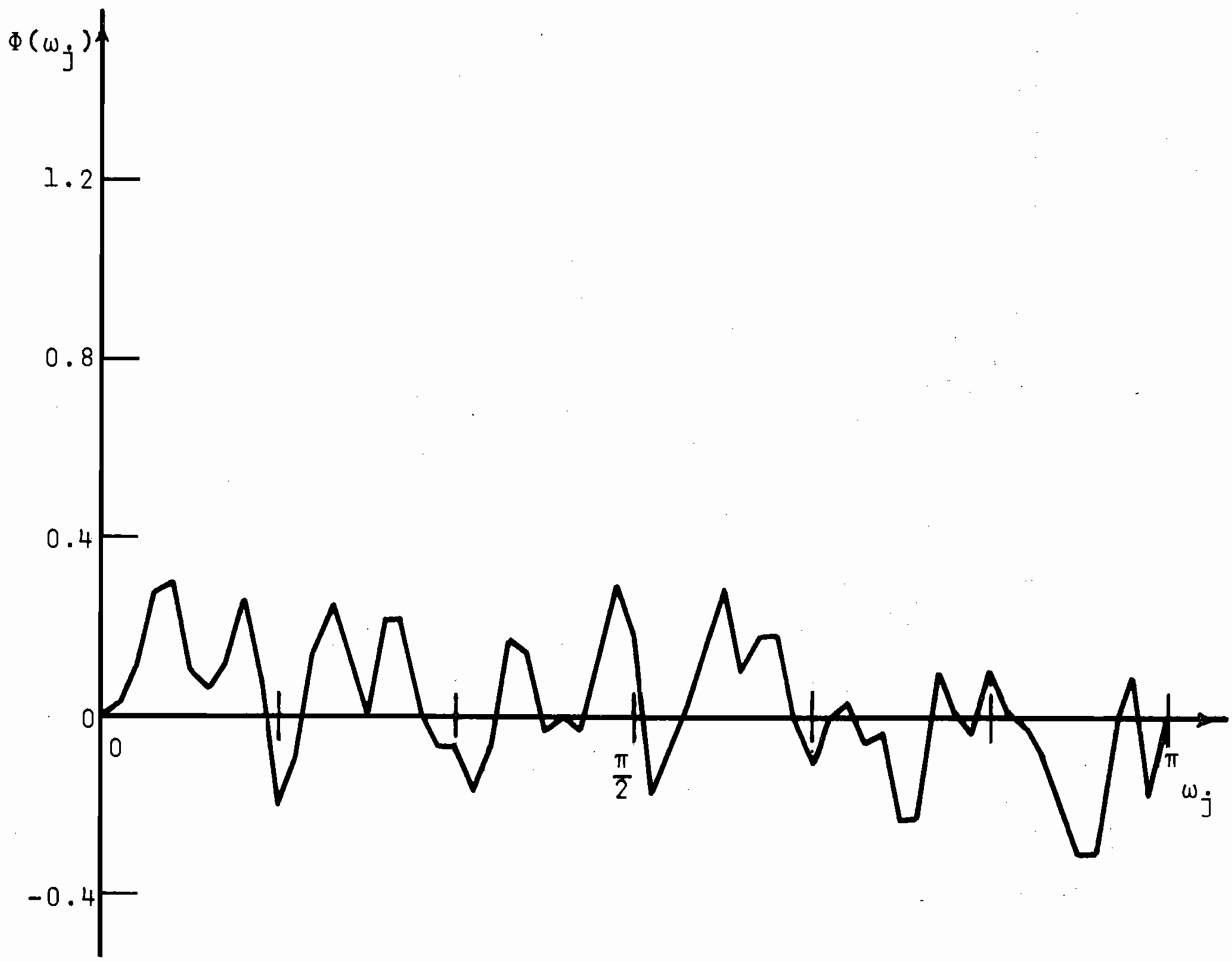

FIG. 13. PHASE DIAGRAM BETWEEN THE STOCHASTIC RESIDUALS OF MONTHLY PRECIPITATIONS AT BLOOMINGTON AND CLINTON, ILLINOIS

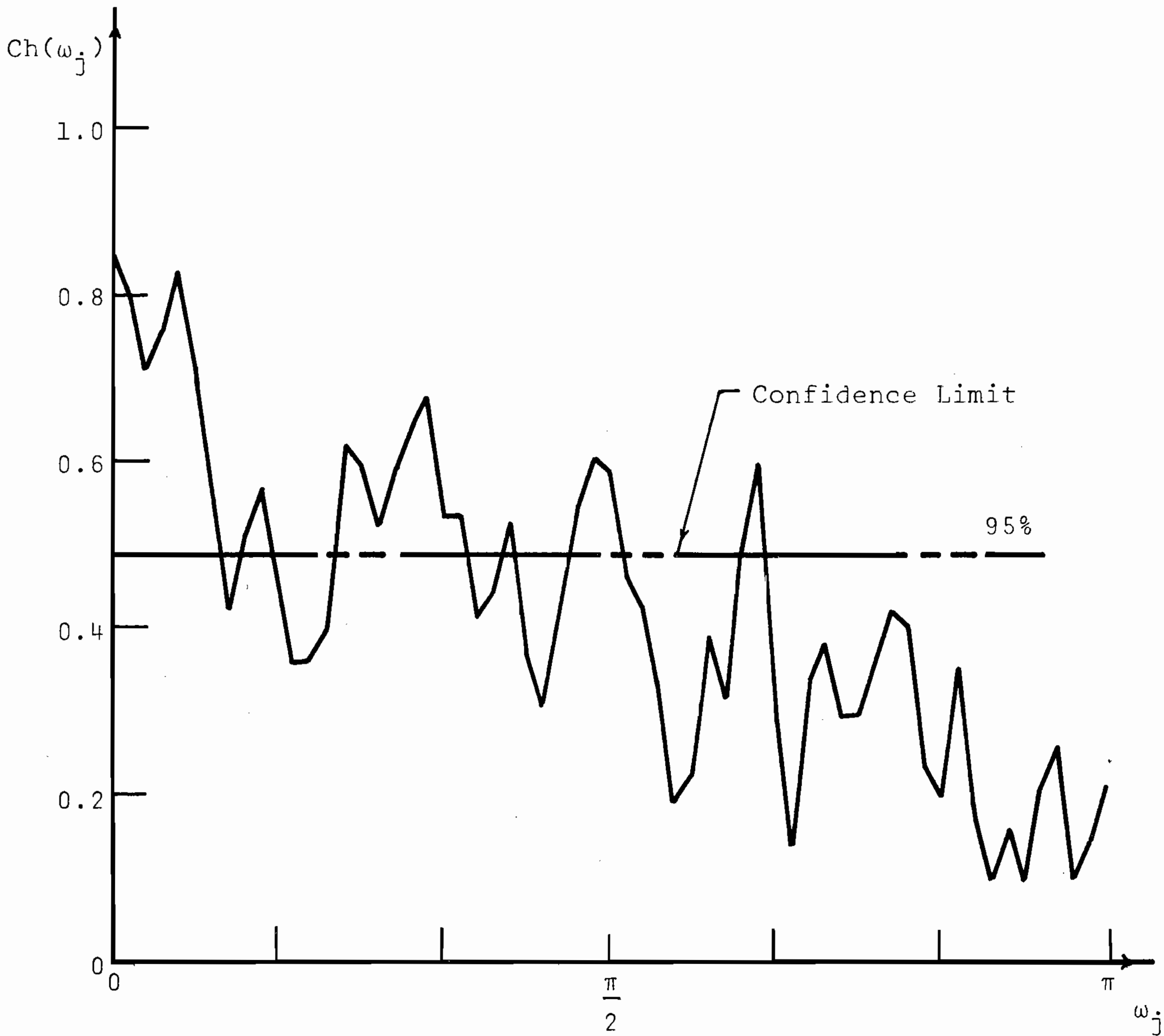

FIG. 14. COHERENCE OF THE STREAMFLOW STOCHASTIC RESIDUALS AT MONTICELLO, ILLINOIS AND THE SERIES $\{\xi_1\}$

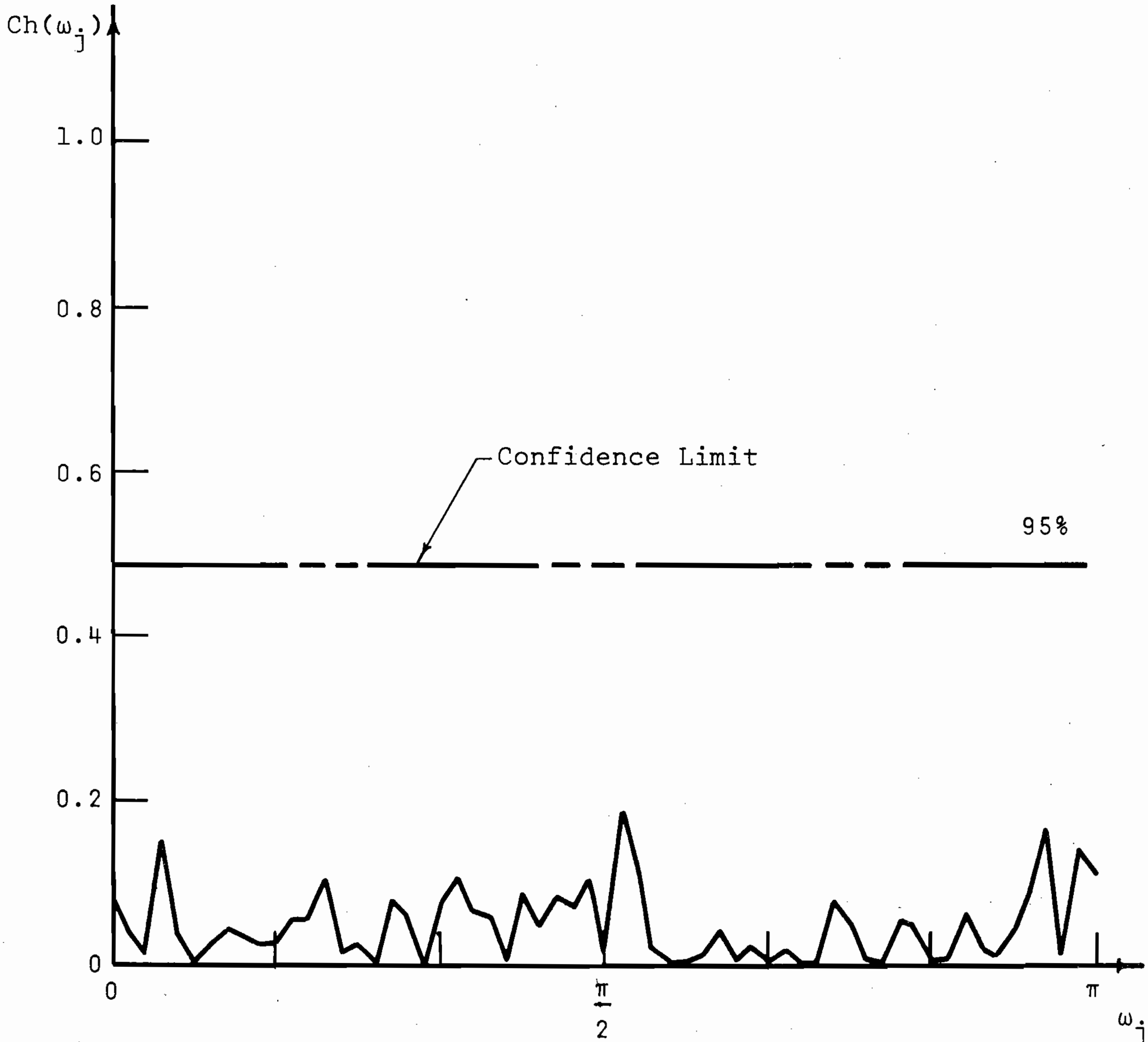

FIG. 15. COHERENCE OF THE STREAMFLOW STOCHASTIC RESIDUALS AT MONTICELLO, ILLINOIS AND THE SERIES $\{\xi_2\}$

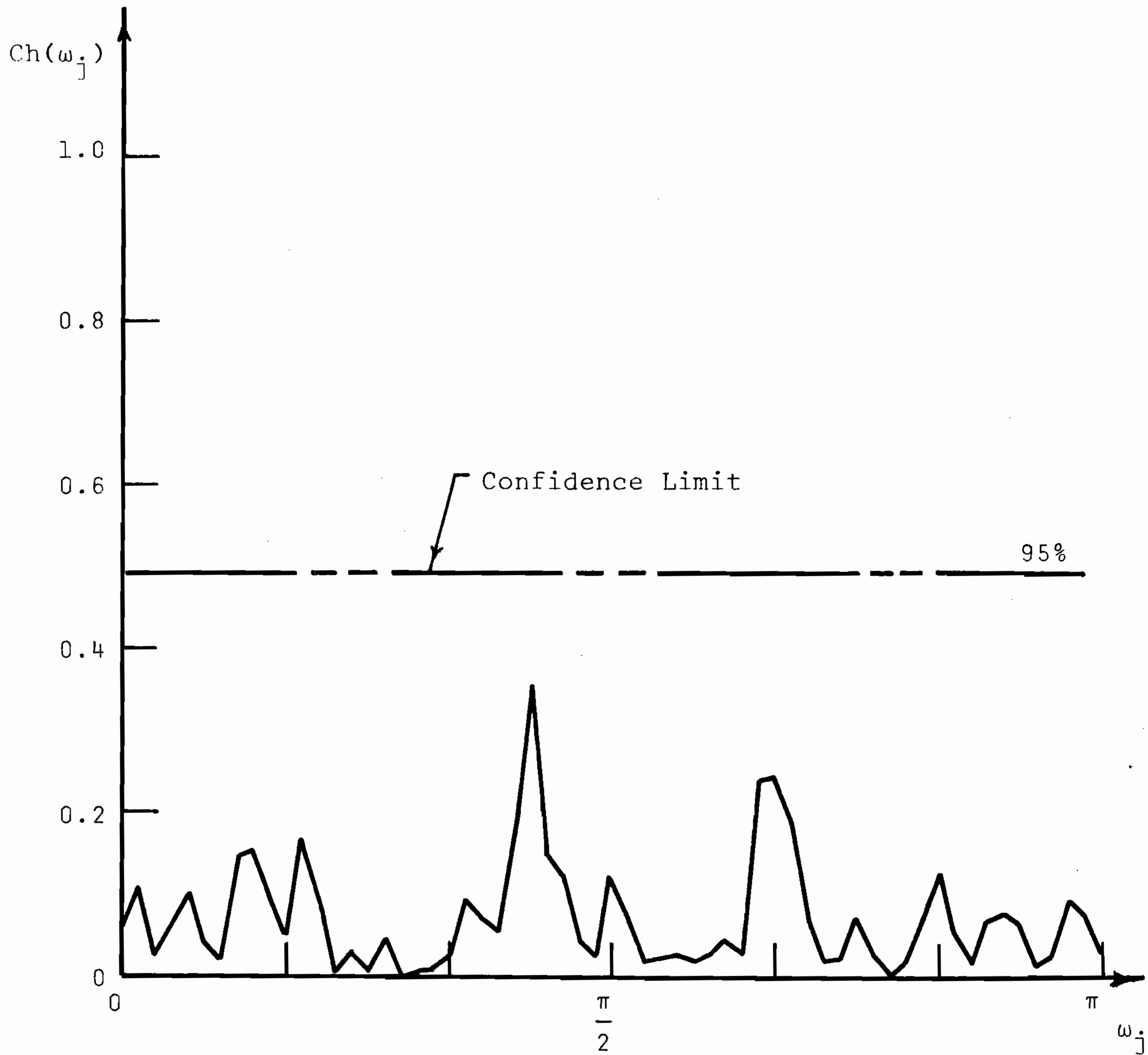

FIG. 16. COHERENCE OF THE STREAMFLOW STOCHASTIC RESIDUALS AT MONTICELLO, ILLINOIS AND THE SERIES $\{\xi_3\}$

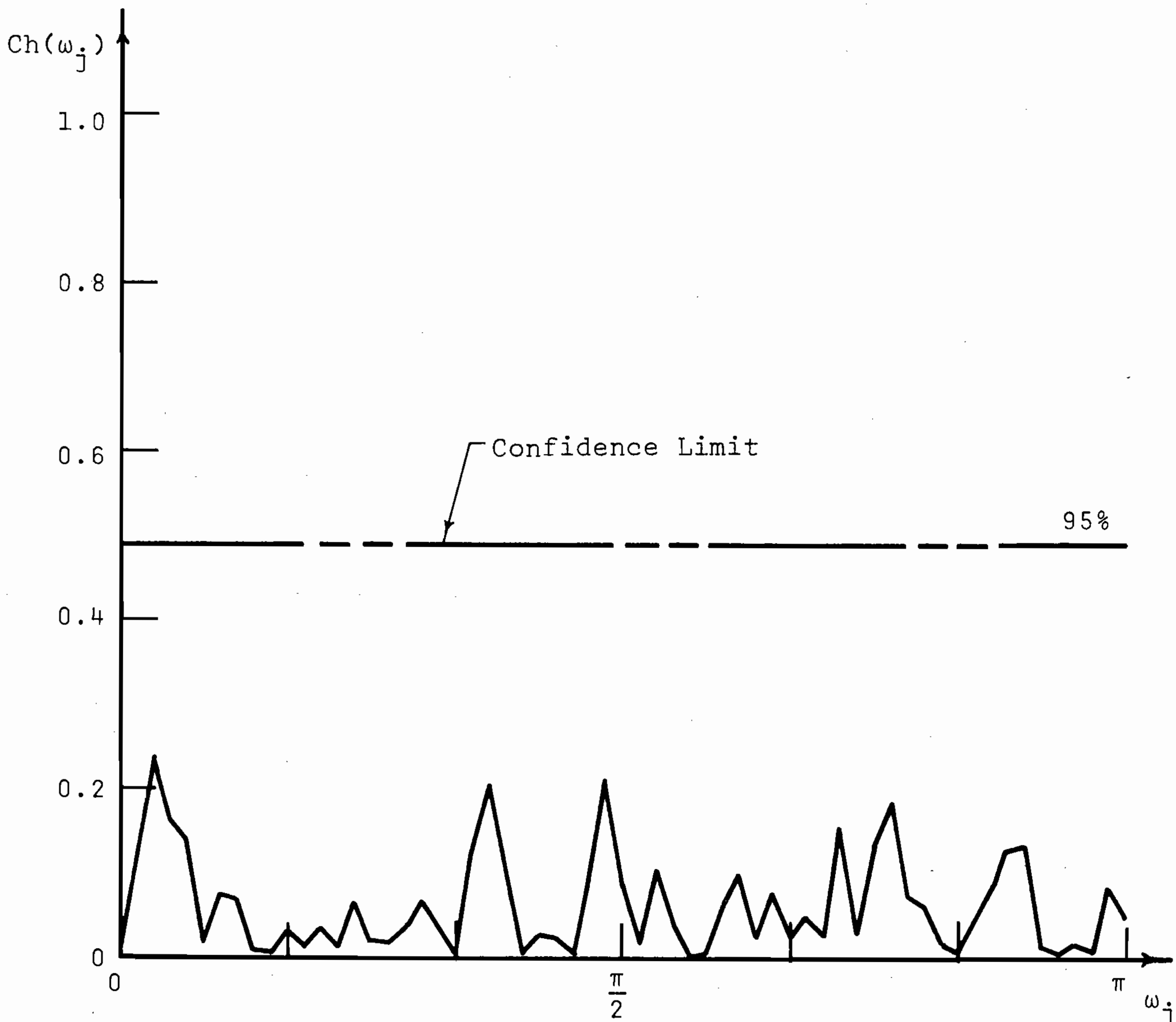

FIG. 17. COHERENCE OF THE STREAMFLOW STOCHASTIC RESIDUALS AT MONTICELLO, ILLINOIS AND THE SERIES $\{\xi_4\}$

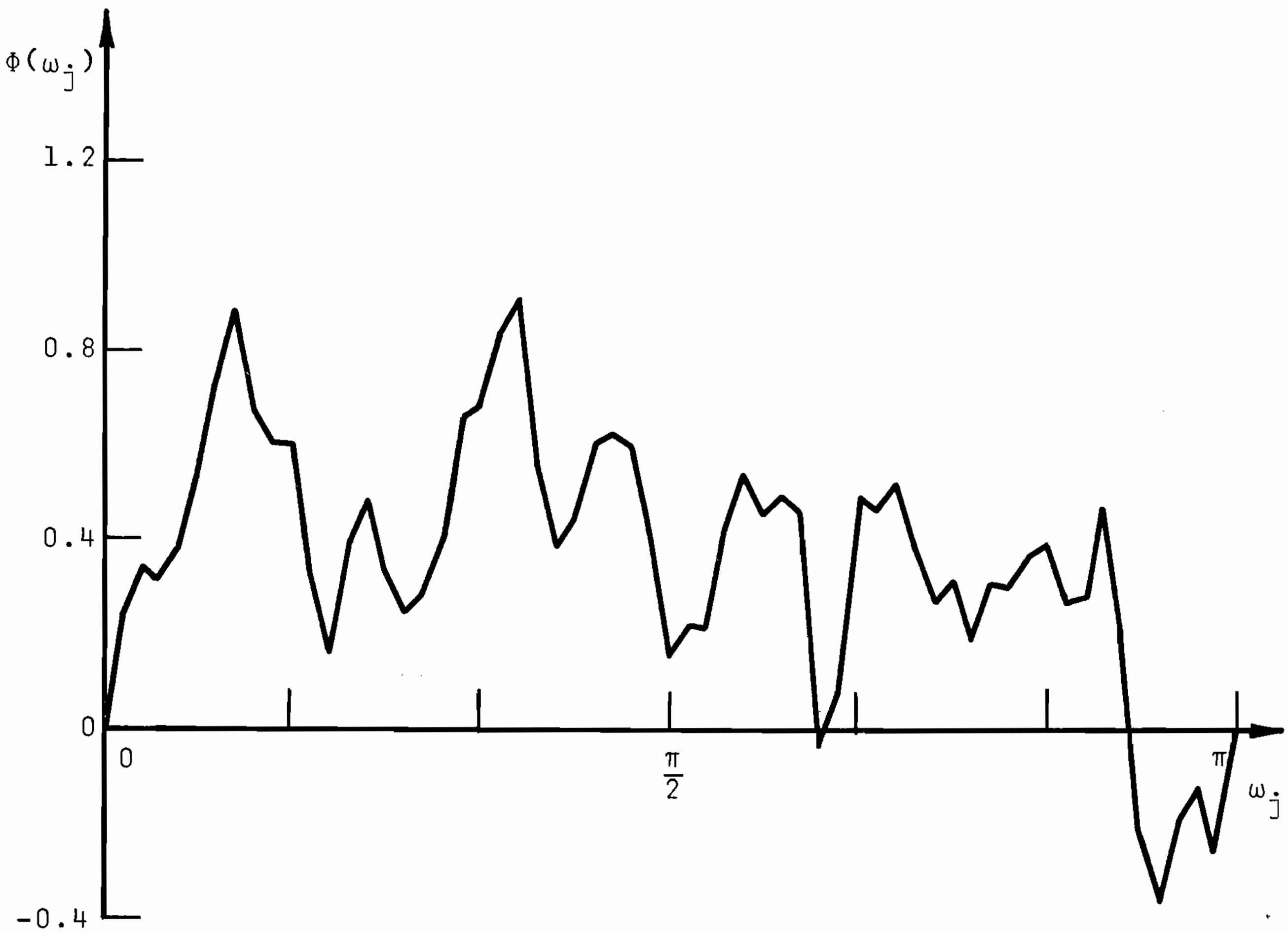

FIG. 18. PHASE DIAGRAM BETWEEN THE STREAMFLOW STOCHASTIC RESIDUALS AT MONTICELLO, ILLINOIS AND THE SERIES $\{\xi_1\}$

APPENDIX: COMPUTER PROGRAMS

Prepared by S. J. Kareliotis

```
C      PROGRAM FOR THE COMPUTATION OF SPECTRA
C
C
C
C
       DIMENSION X(12,110),U(1320),COV(61),OMF(61),BN(61),SN(61)
       DIMENSION TITLE(20)
       DIMENSION M1(10),M2(10)
       EQUIVALENCE (X(1,1),U(1))
C
C      ≥X≥ OR ≥U≥ IS THE HYDROLOGIC VARIABLE
C
     1 READ(5,98,END=2) (TITLE(I),I=1,20)
    98 FORMAT(20A4)
       WRITE(6,97) (TITLE(I),I=1,20)
    97 FORMAT(*1*///4X,20A4)
       READ(5,98) (TITLE(I),I=1,20)
       WRITE(6,96) (TITLE(I),I=1,20)
    96 FORMAT(/4X,20A4)
       READ(5,99) N,N12,NN,M
    99 FORMAT(4I5)
C
C      ≥N≥ IS THE NUMBER OF YEARS (INPUT)
C      ≥N12≥ IS THE NUMBER OF MONTHS, I.E. N12=N*12  (INPUT)
C      ≥NN≥ IS THE MAXIMUM LAG FOR THE COVARIANCES INCREASED BY ONE (INPUT)
C      ≥M≥ IS THE NUMBER OF INTERVALS OF MISSING DATA
C
       IF(M.EQ.0) GO TO 81
       DO 82 I=1,M
    82 READ(5,102) M1(I),M2(I)
   102 FORMAT(2I5)
    81 DO 10 I=1,N
    10 READ(5,100)  (X(J,I),J=1,12)
C      FOR X(J,I) * J=THE MONTH, AND I=THE YEAR
   100 FORMAT(12F5.2)
C
C      COMPUTATION OF AUTOCOVARIANCES
C
       DO 20 K=1,NN
       KK=K-1
       JK=N12-KK
       SX1X2=0.0
       SX1=0.0
       SX2=0.0
       IN=0
       DO 21 I=1,JK
       IKK=I+KK
       IF(M.EQ.0) GO TO 83
       DO 26 J=1,M
       IF(((I.GE.M1(J)).AND.(I.LE.M2(J))).OR.((IKK.GE.M1(J)).AND.(IKK.LE.
      1M2(J)))) GO TO 21
    26 CONTINUE
    83 SX1X2=SX1X2+U(I)*U(IKK)
       SX1=SX1+U(I)
       SX2=SX2+U(IKK)
       IN=IN+1
```

```
   21 CONTINUE
      CJK=IN
   20 COV(K)=(SX1X2/CJK)-(SX1*SX2/(CJK**2))
C     COV(K)  IS THE AUTOCOVARIANCE OF LAG K-1
      WRITE(6,111)
  111 FORMAT(///10X,17HTABLE OF SPECTRUM//5X,6HPERIOD,5X,22HRAW SPECTRAL



     1 ESTIMATES,5X,27HSMOOTHED SPECTRAL ESTIMATES,4X,9HFREQUENCY/)
      N1=NN-1
      CN1=N1
C
C     COMPUTATION OF RAW AND SMOOTHED SPECTRA
C     OME(J) IS THE FREQUENCY
C     BN(J)  IS THE RAW SPECTRUM ESTIMATE AT FREQUENCY OME(J)
C     SN(J)  IS THE SMOOTHED SPECTRUM ESTIMATE AT FREQUENCY OME(J)
C
      DO 11 J=1,NN
      CJ=J-1
      SS=0.0
      AR=CJ*3.14159
      DO 12 I=2,N1
      CI=I-1
   12 SS=SS+COV(I)*COS(CI*AR/CN1)
      BN(J)=(COV(1)+2.*SS+COV(NN)*COS(AR))/(2.*3.14159)
   11 OME(J)=AR/CN1
      SN(1)=0.46*BN(2)+0.54*BN(1)
      J=0
      WRITE(6,112) J,BN(1),SN(1),OME(1)
  112  FORMAT(6X,I3,11X,E12.4,19X,E12.4,8X,E12.4)
      DO 13 I=2,N1
      J=I-1
      JM=I+1
      SN(I)=0.23*BN(J)+0.54*BN(I)+0.23*BN(JM)
   13 WRITE(6,112) J,BN(I),SN(I),OME(I)
      SN(NN)=0.46*BN(N1)+0.54*BN(NN)
      WRITE(6,112) N1,BN(NN),SN(NN),OME(NN)
      GO TO 1
    2 STOP
      END
```

```
C     PROGRAM FOR THE ESTIMATION OF THE COEFFICIENTS OF THE PRIMARY
C     DETERMINISTIC COMPONENT AND THE STOCHASTIC RESIDUALS.
C     THE PRIMARY DETERMINISTIC COMPONENT CONSISTS OF THE 6-MONTH AND
C     12-MONTH PERIODICITIES.
C     THE LEAST-SQUARE METHOD IS USED ACCORDING TO THE PROCEDURE OF
C     R.C.BROWN IN -SMOOTHING,FORCASTING AND PREDICTION OF DISCRETE
C     TIME SERIES- PRENTICE HALL,1962.
C
C
C
C
      DIMENSION X(12,110),U(1320),IYR(110),A(5),F(1320,5),FF(5,5),G(5)
      DIMENSION TITLE(20),TITL1(20),M1(10),M2(10),FE(1320,5),UE(1320)
      EQUIVALENCE (X(1,1),U(1))
C
C     "X" OR "U" IS THE HYDROLOGIC VARIABLE
C
C     THE FUNCTION OF THE PRIMARY DETERMINISTIC COMPONENT IS,
C     FUNC(I)=SUM OF (A(J)*F(I,J)) FOR J=1,...,NM
C     WHERE I DENOTES THE I-TH TIME INTERVAL
      READ(5,95)NM
    1 READ(5,98,END=2) (TITLE(I),I=1,20)
   98 FORMAT(20A4)
      READ(5,98)(TITL1(I),I=1,20)
      WRITE(6,97)(TITLE(I),I=1,20)
   97 FORMAT("1"///4X,20A4)
      WRITE(6,96)(TITL1(I),I=1,20)
   96 FORMAT(/4X,20A4)
      READ(5,98) (TITL1(I),I=1,20)
      WRITE(6,96) (TITL1(I),I=1,20)
   95 FORMAT(I5)


      READ(5,98) (TITL1(I),I=1,20)
      WRITE(6,96) (TITL1(I),I=1,20)
      READ(5,99)N,N12,M
   99 FORMAT(2I5,5X,I5)
C
C     "N" IS THE NUMBER OF YEARS (INPUT)
C     "N12" IS THE NUMBER OF MONTHS, I.E. N12=N*12  (INPUT)
C     "M"   IS THE NUMBER OF INTERVALS OF MISSING DATA
C
C     COMPUTATION OF THE MATRIX F(I,J)
C
      DO 60 I=1,N12
      CI=I
      F(I,1)=1.0
      AR6=3.14159*CI/3.0
      AR12=AR6/2.0
      F(I,2)=COS(AR12)
      F(I,3)=SIN(AR12)
      F(I,4)=COS(AR6)
   60 F(I,5)=SIN(AR6)
```

```
      IF(M.EQ.0) GO TO 21
      DO 201 I=1,M
  201 READ(5,202)M1(I),M2(I)
  202 FORMAT(2I5)
C
C     READ THE DATA
C
   21 DO 10 I=1,N
   10 READ(5,100)  (X(J,I),J=1,12),IYR(I)
C
C     FOR X(J,I) " J=THE MONTH, AND I=THE YEAR
C     IYR(I) IS THE I-TH YEAR OF THE DATA
C
  100 FORMAT(12F5.2,12X,I4)
      IF(M.EQ.0) GO TO 25
      L=0
      DO 22 I=1,N12
      DO 23 J=1,M
      IF((I.GE.M1(J)).AND.(I.LE.M2(J))) GO TO 22
   23 CONTINUE
      L=L+1
      FE(L,1)=1.
      FE(L,2)=F(I,2)
      FE(L,3)=F(I,3)
      FE(L,4)=F(I,4)
      FE(L,5)=F(I,5)
      UE(L)=U(I)
   22 CONTINUE
      CALL VARM(SMEAN,SVAR,UE,FE,G,FF,L,NM)
      GO TO 24
   25 CALL VARM(SMEAN,SVAR,U,F,G,FF,N12,NM)
C
C     COMPUTATION OF THE INVERSE MATRIX OF FF(I,J).
C
   24 CALL INVMAT(NM,FF,0.00001)
C
C
C     COMPUTATION OF THE COEFFICIENTS  A(J)  OF THE PRIMARY DETERMINISTIC
C     COMPONENT
C
      DO 80 I=1,NM
      SS=0.0
      DO 81 J=1,NM
   81 SS=SS+  G(I)*FF(J,I)
      A(I)=SS
   80 WRITE(6,400) I,A(I)
  400 FORMAT(/10X,"A",I1,"=",F15.7)
C
C     COMPUTATION OF THE STOCHASTIC RESIDUALS
C
      WRITE(6,401)(TITLE(I),I=1,20)
  401 FORMAT(///5X,"STOCHASTIC RESIDUALS OF",1X,20A4)
      WRITE(6,337)
```

```
  337 FORMAT(/1X,4HYEAR,3X,3HOCT,4X,3HNOV,4X,3HDEC,4X,3HJAN,4X,3HFEB,4X,
     13HMAR,4X,3HAPR,4X,3HMAY,4X,3HJUN,4X,3HJUL,4X,3HAUG,4X,3HSEP)
      SS1=0.
      SS2=0.
      IF(M.EQ.0) GO TO 26
      DO 84 I=1,L
      DO 85 J=1,NM
   85 UE(I)=UE(I)-A(J)*FE(I,J)
      SS1=SS1+UE(I)
   84 SS2=SS2+UE(I)*UE(I)
      CL=L-NM
      L=0
      DO 27 I=1,N12
      DO 28 J=1,M
      IF((I.GE.M1(J)).AND.(I.LE.M2(J))) GO TO 27
   28 CONTINUE
      L=L+1
      U(I)=UE(L)
   27 CONTINUE
      GO TO 86
   26 DO 82 I=1,N12
      DO 83 J=1,NM
   83 U(I)=U(I)-A(J)*F(I,J)
      SS1=SS1+U(I)
   82 SS2=SS2+U(I)*U(I)
      CL=N12-NM
   86 RMEAN=SS1/CL
      RVAR=SS2/CL
C
      PUNCH 355, (TITLE(I),I=1,20)
  355 FORMAT(*STOCHASTIC RESIDUALS OF*,1X,14A4/6A4)
C
      PUNCH 354, (TITL1(I),I=1,20)
  354 FORMAT(20A4)
      DO 87 I=1,N
C
C     CARDS TO BE PUNCHED FOR THE STOCHASTIC RESIDUALS
C
      PUNCH 356,    (X(J,I),J=1,12),IYR(I)
  356 FORMAT(12F6.2,3X,I4)
   87 WRITE(6,339) IYR(I),(X(J,I),J=1,12)
  339 FORMAT(/1X,I4,12F7.2)
      IF(RMEAN.LT.0.001) RMEAN=0.0
      WRITE(6,955) SMEAN,SVAR,RMEAN,RVAR
  955 FORMAT(/////2X,5HMEAN=E15.7,3X,4HVAR=E15.7//2X,13HMEAN OF  RES.=E15
     1.7,3X,12HVAR OF RES.=E15.7)
      GO TO 1
    2 STOP
      END
```

```
      SUBROUTINE VARM(SMEAN,SVAR,U,F,G,FF,N12,NM)
      DIMENSION U(1320),F(1320,5),G(5),FF(5,5)
C     COMPUTATION OF THE MATRIX FF(I,J) WHICH IS THE PRODUCT OF MATRIX
C     F(L,M) AND ITS TRANSPOSE MATRIX
C
      DO 61 I=1,NM
      DO 62 J=I,NM
      SS=0.0
      DO 63 K=1,N12
   63 SS=SS+F(K,I)*F(K,J)
      FF(J,I)=SS
   62 FF(I,J)=SS
   61 CONTINUE
C
C     COMPUTATION OF THE VECTOR G WHICH IS THE PRODUCT OF THE VECTOR
C     U AND THE MATRIX F
C
      DO 70 I=1,NM
      SS=0.0
      DO 71 K=1,N12
   71 SS=SS+U(K)*F(K,I)
   70 G(I)=SS
C
C     COMPUTATION OF THE SAMPLE MEAN(=SMEAN) AND VARIANCE(=SVAR)
C
      SS1=0.0
      SS2=0.0
      DO 73 I=1,N12
      SS1=SS1+U(I)
   73 SS2=SS2+U(I)*U(I)
      CC=N12
      SMEAN=SS1/CC
      SVAR=(SS2-((SS1*SS1)/CC))/(CC-1.)
      RETURN
      END
```

```
      SUBROUTINE INVMAT(N2,ST22,ERR)
C     COMPUTATION OF THE INVERSE MATRIX OF ST22
C     THE PROGRAM WAS WRITTEN ACCORDING TO THE FLOW CHART GIVEN BY
C     R.BECKETT, AND J.HURT, IN THE BOOK -NUMERICAL CALCULATIONS AND
C     ALGORITHMS- , MCGRAW-HILL,1967,PAGES 119-120.
C
C     N2 IS THE RANK OF MATRIX ST 22
C
      DIMENSION FF(2,4),ST22(2,2)
C
      DO 10 I=1,N2
      DO 10 J=1,N2
   10 FF(I,J)=ST22(I,J)
      N1=N2+1
      N11=N2-1
      NN=2*N2
      DO 101 I=1,N2
      DO 102 J=N1,NN
      IF(J-N2-I) 104,103,104
  103 FF(I,J)=1.0
      GO TO 102
  104 FF(I,J)=0.0
  102 CONTINUE
  101 CONTINUE
      DO 105 K=1,N11



      B=FF(K,K)
      KI=K
      K1=K+1
      DO 106 I=K1,N2
      IF(ABS(B)-ABS(FF(I,K))) 107,106,106
  107 B=FF(I,K)
      KI=I
  106 CONTINUE
C
C     CHECK FOR THE CASE OF SINGULAR MATRIX
C
C     ERR  HAS TO BE SPECIFIED (INPUT)
C
      IF(ABS(B)- ERR    ) 108,109,109
  108 WRITE(6,110) B
  110 FORMAT(/////5X,23HSINGULAR MATRIX WITH B=,E15.7)
      STOP
  109 IF(KI-K) 111,112,111
  111 DO 113 J=K,NN
      B1=FF(K,J)
      FF(K,J)=FF(KI,J)
  113 FF(KI,J)=B1
  112 DO 114 J=K1,NN
  114 FF(K,J)=FF(K,J)/B
      DO 115 I=K1,N2
      DO 116 J=K1,NN
```

```
116 FF(I,J)=FF(I,J)-FF(I,K)*FF(K,J)
115 CONTINUE
105 CONTINUE
    DO 117 J=N1,NN
117 FF(N2,J)=FF(N2,J)/FF(N2,N2)
    DO 118 L=1,N11
    K=N2-N11
    K1=K+1
    DO 119 J=N1,NN
    DO 120 I=K1,N2
120 FF(K,J)=FF(K,J)-FF(K,I)*FF(I,J)
119 CONTINUE
118 CONTINUE
    DO 121 I=1,N2
    DO 122 J=1,N2
    JJ=N2+J
122 ST22(I,J)=FF(I,JJ)
121 CONTINUE
    RETURN
    END
```

```
C       PROGRAM FOR ESTIMATING CROSS-SPECTRA AND PARTIAL CROSS-SPECTRA
C
C
C
C
        COMPLEX ST,STT
        DIMENSION X(12,110,6),U(1320,6),M1(10),M2(10),C(6,6,61),OME(61)
        DIMENSION CO(61,61),SI(61,61),CC(6,6,61),QQ(6,6,61),Q(6,6,61)
        DIMENSION TITLE(20,6,4),ST(6,6,61),STT(6)
        EQUIVALENCE(X(1,1,1),U(1,1))
C
C       X  IS THE RECORD OF PRECIPITATION AND STREAMFLOW
C          RESIDUALS
C
        READ(5,200) NF,NL,NN,NS
  200   FORMAT(4I5)



C       NF IS THE FIRST CALENDAR YEAR FOR ALL THE RECORDS
C       NL IS THE LAST CALENDAR YEAR FOR ALL THE RECORDS
C       NN IS THE MAXIMUN LAG FOR VARIANCES INCREASED BY ONE
C       NS IS THE NUMBER OF TIME SERIES OF THE MULTIPLE TIME SERIES
        N=NL-NF+1
C       N IS THE MAXIMUN LENGTH OF RECORD IN YEARS
        WRITE(6,400)
  400   FORMAT(*1*///5X,*THE DATA FOR THE COMPUTATION OF CROSS-SPECTRA ARE
       1*//)
C       READ THE DATA
C       STREAMFLOW DATA
        READ(5,401)(TITLE(I,1,1),I=1,20),(TITLE(I,1,2),I=1,20)
        WRITE(6,402)(TITLE(I,1,1),I=1,20),(TITLE(I,1,2),I=1,20)
        READ(5,401) (TITLE(I,1,3),I=1,20),(TITLE(I,1,4),I=1,20)
        WRITE(6,402)(TITLE(I,1,3),I=1,20),(TITLE(I,1,4),I=1,20)
  401   FORMAT(20A4/20A4)
  402   FORMAT(//4X,20A4//4X,20A4)
        READ(5,102) NFS,M
C       NFS IS THE FIRST WATER YEAR FOR STREAMFLOW RECORD
C       M IS THE NUMBER OF INTERVALS OF MISSING DATA
        IF(M.EQ.0) GO TO 83
        DO 84 J=1,M
   84   READ(5,102) M1(J),M2(J)
  102   FORMAT(2I5)
   83   K1=NFS-1-NF
        IF(K1.EQ.0) GO TO 85
        DO 86 I=1,K1
        DO 86 J=1,12
   86   X(J,I,1)=999.
   85   KK1=K1+1
        DO 87 J=1,9
   87   X(J,KK1,1)=999.
        NN1=N-1
        DO 88 I=KK1,NN1
        II=I+1
```

```
   88 READ(5,100) (X(J,I,1),J=10,12),(X(JJ,II,1),JJ=1,9)
  100 FORMAT(12F6.2)
      DO 89 J=10,12
   89 X(J,N,1)=999.
      IF(M.EQ.0) GO TO 90
      K12=K1*12+9
      DO 91 I=1,M
      MM1=M1(I)+K12
      MM2=M2(I)+K12
      DO 91 J=MM1,MM2
   91 U(J,1)=999.
   90 CONTINUE
C     PRECIPITATION DATA
      DO 10 IK=2,NS
      READ(5,401)(TITLE(I,IK,1),I=1,20),(TITLE(I,IK,2),I=1,20)
      WRITE(6,402)(TITLE(I,IK,1),I=1,20),(TITLE(I,IK,2),I=1,20)
      READ(5,401)(TITLE(I,IK,3),I=1,20),(TITLE(I,IK,4),I=1,20)
      WRITE(6,402)(TITLE(I,IK,3),I=1,20),(TITLE(I,IK,4),I=1,20)
      READ(5,102) NFS,M
C     NFS IS THE FIRST CALENDAR YEAR FOR PRECIPITATION RECORD
      IF(M.EQ.0) GO TO 81
      DO 82 J=1,M
   82 READ(5,102) M1(J),M2(J)
   81 K1=NFS-NF
      IF(K1.EQ.0) GO TO 70
      DO 71 I=1,K1
      DO 71 J=1,12



   71 X(J,I,IK)=999.
   70 KK1=K1+1
      DO 72 I=KK1,N
   72 READ(5,100)(X(J,I,IK),J=1,12)
      IF(M.EQ.0) GO TO 10
      K12=K1*12
      DO 73 I=1,M
      MM1=M1(I)+K12
      MM2=M2(I)+K12
      DO 73 J=MM1,MM2
   73 U(J,IK)=999.
   10 CONTINUE
C
C     COMPUTATION OF CROSS-COVARIANCES
C
      N12=N*12
      DO 13 L=1,NS
      DO 13 M=1,NS
      DO 30 K=1,NN
      KK=K-1
      JK=N12-KK
      SX1X2=0.0
      SX1=0.
      SX2=0.
      IN=0
```

```
      DO 31 I=1,JK
      IKK=I+KK
      IF((U(I,M).GT.500.).OR.(U(IKK,L).GT.500.)) GO TO 31
      SX1X2=SX1X2+U(I,M)*U(IKK,L)
      SX1=SX1+U(IKK,L)
      SX2=SX2+U(I,M)
      IN=IN+1
   31 CONTINUE
      CJK=IN
   30 C(L,M,K)=(SX1X2-((SX1*SX2)/CJK))/CJK
C
C     C(L,M,I)  IS THE CROSS-COVARIANCE BETWEEN THE L-TH AND M-TH
C               SERIES FOR LAG I-1
C
   13 CONTINUE
      NN1=NN-1
      DO 14 I=2,NN1
      DO 14 L=1,NS
      DO 14 M=1,NS
C
C     NOW C(L,M,I) IS MODIFIED AND REPRESENTS A FUNCTION OF C(L,M,I)
C
   14 C(L,M,I)=2.*C(L,M,I)
      CNN=NN1
      DO 15 I=1,NN
      CI=I-1
      OME(I)=3.14159*CI/CNN
C     OME(J) IS THE FREQUENCY
      DO 15 J=1,NN
      CJ=J-1
C
C     CO(I,J)  IS THE COSINE OF OME(I)*(J-1)
C     SI(I,J)  IS THE SINE OF OME(I)*(J-1)
C
      CO(I,J)=COS(OME(I)*CJ)
   15 SI(I,J)=SIN(OME(I)*CJ)
C



C     COMPUTATION OF CROSS-SPECTRA
C
      DO 16 L=1,NS
      DO 16 M=L,NS
      DO 16 I=1,NN
      SS1=0.0
      SS2=0.0
      DO 17 J=1,NN
      SS1=SS1+(C(L,M,J)+C(M,L,J))*CO(I,J)
   17 SS2=SS2+(C(L,M,J)-C(M,L,J))*SI(I,J)
      CC(L,M,I)=SS1/2.
   16 QQ(L,M,I)=SS2/2.
C
```

```
C      CC(L,M,I)  IS THE RAW ESTIMATE OF CO-SPECTRUM TIMES 2.*3.14159
C                 BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)
C      QQ(L,M,I)  IS THE RAW ESTIMATE OF QUADRATURE SPECTRUM TIMES
C                 2.*3.14159 BETWEEN THE L-TH AND M-TH SERIES AT
C                 FREQUENCY OME(I)
C
      DO 18 L=1,NS
      DO 18 M=L,NS
C
C      C(L,M,I)  IS NOW THE SMOOTHED ESTIMATE OF CO-SPECTRUM BETWEEN
C                THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)
C
C      Q(L,M,I)  IS THE SMOOTHED ESTIMATE OF QUADRATURE SPECTRUM
C                BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)
C      ST(L,M,I)  IS THE ESTIMATED CROSS-SPECTRUM BETWEEN THE L-TH
C                 AND M-TH SERIES AT FREQUENCY OME(I)
C
      C(L,M,1)=(0.54*CC(L,M,1)+0.46*CC(L,M,2))/(2.*3.14159)
      C(L,M,NN)=(0.46*CC(L,M,NN1)+0.54*CC(L,M,NN))/(2.*3.14159)
      Q(L,M,1)=(0.54*QQ(L,M,1))/(2.*3.14159)
      Q(L,M,NN)=(0.54*QQ(L,M,NN))/(2.*3.14159)
      IF(L.NE.M) GO TO 118
      ST(L,L,1)=C(L,L,1)
      ST(L,L,NN)=C(L,L,NN)
      GO TO 119
  118 ST(L,M,1)=CMPLX(C(L,M,1),Q(L,M,1))
      ST(M,L,1)=CONJG(ST(L,M,1))
      ST(L,M,NN)=CMPLX(C(L,M,NN),Q(L,M,NN))
      ST(M,L,NN)=CONJG(ST(L,M,NN))
  119 DO 18 I=2,NN1
      C(L,M,I)=(0.23*CC(L,M,I-1)+0.54*CC(L,M,I)+0.23*CC(L,M,I+1))/(2.*3.
     114159)
      IF(L.NE.M) GO TO 120
      ST(L,L,I)=C(L,L,I)
      GO TO 18
  120 Q(L,M,I)=(0.23*QQ(L,M,I-1)+0.54*QQ(L,M,I)+0.23*QQ(L,M,I+1))/(2.*3.
     114159)
      ST(L,M,I)=CMPLX(C(L,M,I),Q(L,M,I))
      ST(M,L,I)=CONJG(ST(L,M,I))
   18 CONTINUE
C
C      COMPUTATION OF PARTIAL CROSS-SPECTRA
C
      NNS=NS-1
      DO 60 L=1,NNS
      K=L+1
      DO 61 M=K,NS
      WRITE(6,403)(TITLE(I,L,1),I=1,20),(TITLE(I,M,1),I=1,20)
  403 FORMAT("1"///5X,"CROSS-SPECTRUM BETWEEN"/5X,20A4/1X,"AND",
     11X,20A4)
   61 CALL PART(ST,NS,NN,OME)
      DO 62 I=1,NN
      DO 62 IL=1,L
      DO 63 LI=1,NS
```

```
63 STT(LI)=ST(LI,1,I)
   DO 64 LLL=2,NS
   LLK=LLL-1
   DO 64 LLM=1,NS
64 ST(LLM,LLK,I)=ST(LLM,LLL,I)
   DO 66 LI=1,NS
66 ST(LI,NS,I)=STT(LI)
   DO 67 LI=1,NS
67 STT(LI)=ST(1,LI,I)
   DO 68 LLL=2,NS
   LLK=LLL-1
   DO 68 LLM=1,NS
68 ST(LLK,LLM,I)=ST(LLL,LLM,I)
   DO 69 LI=1,NS
69 ST(NS,LI,I)=STT(LI)
62 CONTINUE
60 CONTINUE
   STOP
   END
```

```
      SUBROUTINE PART(ST,NS,NN,OME)
C     THIS SUBROUTINE COMPUTES THE PARTIAL COHERENCE, PHASE ANGLE,
C     AND GAIN BETWEEN FIRST AND SECOND TIME SERIES
      COMPLEX ST,STT,ST12,ST21,ST22,SN,COH,GAIN
      DIMENSION ST(6,6,61),STT(6),ST12(2,4),ST21(4,4),ST22(4,4),OME(61),
     1SN(2,2,61)
      DIMENSION A(4,4),AA(4,4),B(4,4)
C
C     >NN> IS THE MAXIMUM LAG FOR THE COVARIANCES INCREASED BY ONE (INPUT)
C     ST(L,M,I)  IS THE ESTIMATED CROSS-SPECTRUM BETWEEN THE L-TH
C                AND M-TH SERIES AT FREQUENCY OME(I) (INPUT)
C     OME(J) IS THE FREQUENCY(INPUT)
      NNS=NS-2
      DO 19 I=1,NN
      DO 20 L=1,NNS
      LL=L+2
      DO 21 M=1,2
      ST12(M,L)=ST(M,LL,I)
   21 ST21(L,M)=ST(LL,M,I)
      DO 22 M=1,NNS
      LM=M+2
   22 ST22(L,M)=ST(LL,LM,I)
   20 CONTINUE
      DO 14 I=1,NNS
      DO 14 J=1,NNS
      A(I,J)=ST22(I,J)
      AA(I,J)=ST22(I,J)
   14 B(I,J)=AIMAG(ST22(I,J))
      CALL INVMAT(NNS,AA,0.00001)
      CALL MULT(AA,B,NNS,NNS,NNS)
      CALL MULT(B,AA,NNS,NNS,NNS)
      DO 15 I=1,NNS
      CALL INVMAT(NNS,A,0.00001)
      CALL MULT(AA,A,NNS,NNS,NNS)
      DO 13 I=1,NNS
      DO 13 J=1,NNS
      AA(I,J)=-AA(I,J)
   13 ST22(I,J)=CMPLX(A(I,J),AA(I,J))
      CALL CMULT(ST12,ST22,2,NNS,NNS)
      CALL CMULT(ST12,ST21,2,NNS,2)
      DO 10 L=1,2
      DO 10 M=1,2
   10 SN(L,M,I)=ST(L,M,I)-ST12(L,M)
C
C     SN(L,M,I)  IS THE PARTIAL CROSS-SPECTRUM AT FREQUENCY OME(I)
C                BETWEEN THE L-TH SERIES     AND M-TH SERIES
C
   19 CONTINUE
      DO 11 L=1,2
      DO 11 M=1,2
      IF((L.EQ.2).AND.(M.EQ.1)) GO TO 12
      WRITE(6,112) L,M
```

```
  112 FORMAT(//5X,10HFOR SERIES,1X,I1,1X,3HAND,1X,I1)
      WRITE(6,111)
  111 FORMAT(//2X,6HPERIOD,3X,9HFREQUENCY,10X,8HSPECTRUM,17X,9HCOHERENCE
     1,12X,5HPHASE,15X,4HGAIN/22X,11HCO-SPECTRUM,2X,10HQUADRATURE,6X,4HR
     2EAL,6X,9HIMAGINARY,20X,4HREAL,2X,9HIMAGINARY/)
      DO 30 I=1,NN
      COH=(SN(L,M,I)*CONJG(SN(L,M,I)))/(SN(L,L,I)*SN(M,M,I))
C     COH IS THE COHERENCE
      PHA=ATAN(AIMAG(SN(L,M,I))/REAL(SN(L,M,I)))
C     PHA IS THE PHASE ANGLE
      GAIN=SN(L,M,I)/SN(M,M,I)
      KK=I-1
      A1=REAL(SN(L,M,I))
C     A1 IS THE CO-SPECTRUM
C
      A2=AIMAG(SN(L,M,I))
C     A2 IS THE QUADRATURE SPECTRUM
C
      B1=REAL(COH)
C     B1 IS THE COHERENCE
C
      B2=AIMAG(COH)
C     B2  IS A CHECK OF THE COMPUTATIONS AND MUST BE EQUAL TO ZERO
C
      C1=REAL(GAIN)
C     C1 IS THE REAL PART OF THE GAIN
C
      C2=AIMAG(GAIN)
C     C2 IS THE IMAGINARY PART OF THE GAIN
C
   30 WRITE(6,113) KK,OME(I),A1,A2,B1,B2,PHA,C1,C2
  113 FORMAT(4X,I2,4X,F7.5,3X,E10.4,2X,E10.4,3X,E10.4,3X,E10.4,3X,E10.4,
     13X,E10.4,2X,E10.4)
   12 CONTINUE
   11 CONTINUE
      DO 62 I=1,NN
      DO 63 LI=1,NS
   63 STT(LI)=ST(LI,2,I)
      DO 64 LLL=3,NS
      LLK=LLL-1
      DO 64 LLM=1,NS
   64 ST(LLM,LLK,I)=ST(LLM,LLL,I)
      DO 66 LI=1,NS
   66 ST(LI,NS,I)=STT(LI)
      DO 67 LI=1,NS
   67 STT(LI)=ST(2,LI,I)
      DO 68 LLL=3,NS
      LLK=LLL-1
      DO 68 LLM=1,NS
   68 ST(LLK,LLM,I)=ST(LLL,LLM,I)
      DO 69 LI=1,NS
   69 ST(NS,LI,I)=STT(LI)
   62 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MULT(A,B,L,M,N)
C     TO MULTIPLY MATRICES A AND B AND STORE THE PRODUCT IN A
C     THE ELEMENTS OF THE MATRICES ARE COMPLEX
C
C
C
C
C     L IS THE NUMBER OF ROWS IN A
C     M IS THE NUMBER OF COLUMNS IN A
C     N IS THE NUMBER OF COLUMNS IN B
      COMPLEX A,B,C
      DIMENSION A(4,4),B(4,4),C(4,4)
      DO 1 I=1,L
      DO 1 J=1,N
    1 C(I,J)=0.
      DO 10 I=1,L
      DO 10 J=1,N
      DO 10 K=1,M
   10 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      DO 11 I=1,L
      DO 11 J=1,N
   11 A(I,J)=C(I,J)
      RETURN
      END
```

```
      SUBROUTINE CMULT(A,B,L,M,N)
      DIMENSION A(2,4),B(4,4),C(2,4)
      DO 1 I=1,L
      DO 1 J=1,N
    1 C(I,J)=0.
      DO 10 I=1,L
      DO 10 J=1,N
      DO 10 K=1,M
   10 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      DO 11 I=1,L
      DO 11 J=1,N
   11 A(I,J)=C(I,J)
      RETURN
      END
```

```
C      PROGRAM FOR THE  ESTIMATION OF CROSS-SPECTRA
C
C
C
C
      COMPLEX ST,STT
      COMPLEX COH,GAIN
      DIMENSION X(12,110,6),U(1320,6),M1(10),M2(10),C(6,6,61),OMF(61)
      DIMENSION CO(61,61),SI(61,61),CC(6,6,61),QQ(6,6,61),Q(6,6,61)
      DIMENSION TITLE(20,6,4),ST(6,6,61),STT(6)
      EQUIVALENCE(X(1,1,1),U(1,1))
C
C      X  IS THE RECORD OF PRECIPITATION AND STREAMFLOW
C         RESIDUALS
C
      READ(5,200) NF,NL,NN,NS
  200 FORMAT(4I5)
C     NF IS THE FIRST CALENDAR YEAR FOR ALL THE RECORDS
C     NL IS THE LAST CALENDAR YEAR FOR ALL THE RECORDS
C     NN IS THE MAXIMUN LAG FOR VARIANCES INCREASED BY ONE
C     NS IS THE NUMBER OF TIME SERIES OF THE MULTIPLE TIME SERIES
      N=NL-NF+1
C     N IS THE MAXIMUN LENGTH OF RECORD IN YEARS
      WRITE(6,400)
  400 FORMAT("1"///5X,"THE DATA FOR THE COMPUTATION OF CROSS-SPECTRA ARE
     1"//)
C     READ THE DATA
C     STREAMFLOW DATA
      READ(5,401)(TITLE(I,1,1),I=1,20),(TITLE(I,1,2),I=1,20)
  401 FORMAT(20A4/20A4)
      WRITE(6,402)(TITLE(I,1,1),I=1,20),(TITLE(I,1,2),I=1,20)
  402 FORMAT(//4X,20A4//4X,20A4)
      READ(5,401) (TITLE(I,1,3),I=1,20),(TITLE(I,1,4),I=1,20)
      WRITE(6,402)(TITLE(I,1,3),I=1,20),(TITLE(I,1,4),I=1,20)
      READ(5,102) NFS,M
C     NFS IS THE FIRST WATER YEAR FOR STREAMFLOW RECORD
C     M IS THE NUMBER OF INTERVALS OF MISSING DATA
      IF(M.EQ.0) GO TO 83
      DO 84 J=1,M
   84 READ(5,102) M1(J),M2(J)
  102 FORMAT(2I5)
   83 K1=NFS-1-NF
      IF(K1.EQ.0) GO TO 85
      DO 86 I=1,K1
      DO 86 J=1,12
   86 X(J,I,1)=999.
   85 KK1=K1+1
      DO 87 J=1,9
   87 X(J,KK1,1)=999.
      NN1=N-1
      DO 88 I=KK1,NN1
      II=I+1
   88 READ(5,100) (X(J,I,1),J=10,12),(X(JJ,II,1),JJ=1,9)
  100 FORMAT(12F6.2)
```

```
      DO 89 J=10,12
   89 X(J,N,1)=999.
      IF(M.EQ.0) GO TO 90
      K12=K1*12+9
      DO 91 I=1,M
      MM1=M1(I)+K12
      MM2=M2(I)+K12
      DO 91 J=MM1,MM2




   91 U(J,1)=999.
   90 CONTINUE
C     PRECIPITATION DATA
      IK=2
      READ(5,401)(TITLE(I,IK,1),I=1,20),(TITLE(I,IK,2),I=1,20)
      WRITE(6,402)(TITLE(I,IK,1),I=1,20),(TITLE(I,IK,2),I=1,20)
      READ(5,102) NFS,M
C     NFS IS THE FIRST CALENDAR YEAR FOR PRECIPITATION RECORD
      IF(M.EQ.0) GO TO 81
      DO 82 J=1,M
   82 READ(5,102) M1(J),M2(J)
   81 K1=NFS-NF
      IF(K1.EQ.0) GO TO 70
      DO 71 I=1,K1
      DO 71 J=1,12
   71 X(J,I,IK)=999.
   70 KK1=K1+1
      DO 72 I=KK1,N
   72 READ(5,100)(X(J,I,IK),J=1,12)
      IF(M.EQ.0) GO TO 10
      K12=K1*12
      DO 73 I=1,M
      MM1=M1(I)+K12
      MM2=M2(I)+K12
      DO 73 J=MM1,MM2
   73 U(J,IK)=999.
   10 CONTINUE
      NN1=NN-1
      CNN=NN1
      DO 15 I=1,NN
      CI=I-1
      OME(I)=3.14159*CI/CNN
C     OME(J) IS THE FREQUENCY
      DO 15 J=1,NN
      CJ=J-1
C
C     CO(I,J)  IS THE COSINE OF OME(I)*(J-1)
C     SI(I,J)  IS THE SINE OF OME(I)*(J-1)
C
      CO(I,J)=COS(OME(I)*CJ)
   15 SI(I,J)=SIN(OME(I)*CJ)
```

```
C
C     COMPUTATION OF CROSS-COVARIANCES
C
      N12=N*12
      DO 13 L=1,NS
      DO 13 M=1,NS
      DO 30 K=1,NN
      KK=K-1
      JK=N12-KK
      SX1X2=0.0
      SX1=0.
      SX2=0.
      IN=0
      DO 31 I=1,JK
      IKK=I+KK
      IF((U(I,M).GT.500.).OR.(U(IKK,L).GT.500.)) GO TO 31
      SX1X2=SX1X2+U(I,M)*U(IKK,L)
      SX1=SX1+U(IKK,L)
      SX2=SX2+U(I,M)
      IN=IN+1
   31 CONTINUE


      CJK=IN
   30 C(L,M,K)=(SX1X2-((SX1*SX2)/CJK))/CJK
C
C     C(L,M,I)  IS THE CROSS-COVARIANCE BETWEEN THE L-TH AND M-TH
C               SERIES FOR LAG I-1
C
   13 CONTINUE
      DO 14 I=2,NN1
      DO 14 L=1,NS
      DO 14 M=1,NS
C
C     NOW C(L,M,I) IS MODIFIED AND REPRESENTS A FUNCTION OF C(L,M,I)
C
   14 C(L,M,I)=2.*C(L,M,I)
C
C     COMPUTATION OF CROSS-SPECTRA
C
      DO 16 L=1,NS
      DO 16 M=L,NS
      DO 16 I=1,NN
      SS1=0.0
      SS2=0.0
      DO 17 J=1,NN
      SS1=SS1+(C(L,M,J)+C(M,L,J))*CO(I,J)
   17 SS2=SS2+(C(L,M,J)-C(M,L,J))*SI(I,J)
      CC(L,M,I)=SS1/2.
   16 QQ(L,M,I)=SS2/2.
```

```
C
C      CC(L,M,I)  IS THE RAW ESTIMATE OF CO-SPECTRUM TIMES 2.*3.14159
C                 BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)
C      QQ(L,M,I)  IS THE RAW ESTIMATE OF QUADRATURE SPECTRUM TIMES
C                 2.*3.14159 BETWEEN THE L-TH AND M-TH SERIES AT
C                 FREQUENCY OME(I)
C
       DO 18 L=1,NS
       DO 18 M=L,NS
C
C      C(L,M,I)  IS NOW THE SMOOTHED ESTIMATE OF CO-SPECTRUM BETWEEN
C                THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)
C
C      Q(L,M,I)  IS THE SMOOTHED ESTIMATE OF QUADRATURE SPECTRUM
C                BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)
C      ST(L,M,I)  IS THE ESTIMATED CROSS-SPECTRUM BETWEEN THE L-TH
C                 AND M-TH SERIES AT FREQUENCY OME(I)
C
       C(L,M,1)=(0.54*CC(L,M,1)+0.46*CC(L,M,2))/(2.*3.14159)
       C(L,M,NN)=(0.46*CC(L,M,NN1)+0.54*CC(L,M,NN))/(2.*3.14159)
       Q(L,M,1)=(0.54*QQ(L,M,1))/(2.*3.14159)
       Q(L,M,NN)=(0.54*QQ(L,M,NN))/(2.*3.14159)
   118 ST(L,M,1)=CMPLX(C(L,M,1),Q(L,M,1))
       ST(L,M,NN)=CMPLX(C(L,M,NN),Q(L,M,NN))
       DO 18 I=2,NN1
       C(L,M,I)=(0.23*CC(L,M,I-1)+0.54*CC(L,M,I)+0.23*CC(L,M,I+1))/(2.*3.
      114159)
   120 Q(L,M,I)=(0.23*QQ(L,M,I-1)+0.54*QQ(L,M,I)+0.23*QQ(L,M,I+1))/(2.*3.
      114159)
       ST(L,M,I)=CMPLX(C(L,M,I),Q(L,M,I))
    18 CONTINUE
       DO 95 L=1,NS
       DO 95 M=L,NS
       WRITE(6,403)(TITLE(I,L,1),I=1,20),(TITLE(I,M,1),I=1,20)
   403 FORMAT("1"///5X,"CROSS-SPECTRUM BETWEEN"/5X,20A4/1X,"AND",
      11X,20A4)
       WRITE(6,111)
   111 FORMAT(//2X,6HPERIOD,3X,9HFREQUENCY,10X,8HSPECTRUM,17X,9HCOHERENCE
      1,12X,5HPHASE,15X,4HGAIN/22X,11HCO-SPECTRUM,2X,10HQUADRATURE,6X,4HR
      2EAL,6X,9HIMAGINARY,20X,4HREAL,2X,9HIMAGINARY/)
       DO 95 I=1,NN
       KK=I-1
       COH=(ST(L,M,I)*CONJG(ST(L,M,I)))/(ST(L,L,I)*ST(M,M,I))
C      COH IS THE COHERENCE
       PHA=ATAN(AIMAG(ST(L,M,I))/REAL(ST(L,M,I)))
C      PHA IS THE PHASE ANGLE
       GAIN=ST(L,M,I)/ST(M,M,I)
       A1=REAL(ST(L,M,I))
C      A1 IS THE CO-SPECTRUM
C
```

```
      A2=AIMAG(ST(L,M,I))
C     A2 IS THE QUADRATURE SPECTRUM
C
      B1=REAL(COH)
C     B1 IS THE COHERENCE
      B2=AIMAG(COH)
C     B2  IS A CHECK OF THE COMPUTATIONS AND MUST BE EQUAL TO ZERO
C
      C1=REAL(GAIN)
C     C1 IS THE REAL PART OF THE GAIN
C
      C2=AIMAG(GAIN)
C     C2 IS THE IMAGINARY PART OF THE GAIN
C
   95 WRITE(6,113) KK,OME(I),A1,A2,B1,B2,PHA,C1,C2
  113 FORMAT(4X,I2,4X,F7.5,3X,E10.4,2X,E10.4,3X,E10.4,3X,E10.4,3X,E10.4,
     13X,E10.4,2X,E10.4)
      STOP
      END
```

```
C     PROGRAM FOR THE COMPUTATION OF THE REGRESSION COEFFICIENT
C
C
C
C
      DIMENSION X(12,80),U(960),Y(12,80),W(960),IYR(80),TITLE(20,3),M1(3
     1),M2(3),Z(12,80),V(960)
      DIMENSION C(2,2)
      EQUIVALENCE(X(1,1),U(1)),(Y(1,1),W(1)),(Z(1,1),V(1))
      READ(5,99) (TITLE(I,1),I=1,20),(TITLE(J,2),J=1,20)
   99 FORMAT(20A4/20A4)




      WRITE(6,98) (TITLE(I,1),I=1,20),(TITLE(J,2),J=1,20)
   98 FORMAT(*1*/2X,*REGRESSION COEFFICIENT FOR*,1X,20A4/20A4)
      READ(5,97) M
   97 FORMAT(5X,I5)
      DO 10 I=1,M
   10 READ(5,96) M1(I),M2(I)
   96 FORMAT(2I5)
      DO 11 I=1,59
   11 READ(5,100) (X(J,I),J=1,12),IYR(I)
  100 FORMAT(12F6.2,3X,I4)
      READ(5,95) (TITLE(I,1),I=1,20),(TITLE(J,2),J=1,20)
   95 FORMAT(20A4/20A4)
      WRITE(6,93) (TITLE(I,1),I=1,20),(TITLE(J,2),J=1,20)
   93 FORMAT(2X,*AND*,20A4/20A4)
      DO 12 I=1,59
   12 READ(5,100) (Z(J,I),J=1,12),IYR(I)
      DO 13 I=1,M
      KK=M1(I)
      KKK=M2(I)
      DO 13 J=KK,KKK
   13 U(J)=999.
      SS11=0.0
      SS22=0.0
      SS12=0.0
      SS21=0.0
      DO 20 J=1,705
      I=J+2
      L=I+1
      IF((U(I).GT.400.).OR.(U(L).GT.400.)) GO TO 20
      SS11=SS11+U(I)*U(I)
      SS12=SS12+U(I)*V(J)
      SS22=SS22+V(J)*V(J)
   20 CONTINUE
      C(1,1)=SS11
      C(1,2)=SS12
      C(2,1)=SS12
      C(2,2)=SS22
```

```
      CALL INVMAT(2,C,0,00001)
      SS11=0.0
      SS21=0.0
      DO 21 I=1,705
      J=I+2
      L=J+1
      IF((U(J).GT.400.).OR.(U(L).GT.400.)) GO TO 21
      SS11=SS11+U(J)*U(L)
      SS21=SS21+V(I)*U(L)
   21 CONTINUE
      A1=C(1,1)*SS11+C(1,2)*SS21
      A2=C(2,1)*SS11+C(2,2)*SS21
      WRITE(6,101) A1,A2
  101 FORMAT(/5X,*A1=*,E15,7,5X,*A2=*,E15,7)
      WRITE(6,337)
  337 FORMAT(/1X,4HYEAR,3X,3HOCT,4X,3HNOV,4X,3HDEC,4X,3HJAN,4X,3HFEB,4X,
     13HMAR,4X,3HAPR,4X,3HMAY,4X,3HJUN,4X,3HJUL,4X,3HAUG,4X,3HSEP)
      NI=0
      SS=0.0
      SSS=0.0
      DO 61 I=1,705
      J=I+3
      L=I+2
      IF((U(J).GT.400.).OR.(U(L).GT.400.)) GO TO 22
      W(J)=U(J)-A1*U(L)-A2*V(I)



      SS=SS+W(J)
      SSS=SSS+W(J)*W(J)
      NI=NI+1
      GO TO 61
   22 W(J)=999.
   61 CONTINUE
      WRITE(6,338) IYR(1),(W(I),I=4,12)
  338 FORMAT(/1X,I4,21X,9F7,2)
      PUNCH 357, (W(I),I=4,12)  ,IYR(1)
  357 FORMAT(18X,9F6,2,3X,I4)
      DO 63 I=2,59
      WRITE(6,339) IYR(I),(Y(J,I),J=1,12)
  339 FORMAT(/1X,I4,12F7,2)
   63 PUNCH 356, (Y(J,I),J=1,12),IYR(I)
  356 FORMAT(12F6,2,3X,I4)
      CN=NI
      RMEAN=SS/CN
      RVAR=SSS/CN
      IF(RMEAN,LT,0,001) RMEAN=0,0
      WRITE(6,355) RMEAN,RVAR
  355 FORMAT(/////2X,5HMEAN=E15,7,3X,4HVAR=E15,7)
      STOP
      END
```

```
C      PROGRAM FOR THE DETERMINATION OF EIGENVALUES AND EIGENVECTORS
C         OF THE DISPERSION MATRIX
C
C
C
C
      REAL*8 X(12,80,4),U(960,4),R(16),VA(4),DUM1(10),T(4,4)
      DIMENSION TITLE(20,4,2)
      EQUIVALENCE(X(1,1,1),U(1,1))
C
C     U OR X  ARE THE PRECIPITATION DATA
C
      READ(5,99) L,N
   99 FORMAT(2I5)
C     L  IS THE NUMBER OF STATIONS
C     N  IS THE NUMBER OF YEARS OF MONTHLY DATA
      WRITE(6,400)
  400 FORMAT("1"///5X,"THE DATA FOR THE COMPUTATION ARE"//)
      DO 10 K=1,L
      READ(5,401)(TITLE(I,K,1),I=1,20),(TITLE(I,K,2),I=1,20)
  401 FORMAT(20A4/20A4)
      WRITE(6,402)(TITLE(I,K,1),I=1,20),(TITLE(I,K,2),I=1,20)
  402 FORMAT(//4X,20A4//4X,20A4)
   22 DO 10 I=1,N
   10 READ(5,100) (X(J,I,K),J=1,12)
  100 FORMAT(12F6.2)
      WRITE(6,101)
  101 FORMAT("1"///8X,"DISPERSION MATRIX"//)
      N12=N*12
      CN=N12
      DO 11 K=1,L
      DO 13 M=K,L
      SUM=0.0
      DO 12 I=1,N12
   12 SUM=SUM+U(I,K)*U(I,M)
      T(K,M)=SUM/CN
   13 T(M,K)=T(K,M)
   11 WRITE(6,102) (T(K,M),M=1,L)
  102 FORMAT(/8X,4(F8.5,2X))



      CALL EIGENZ(T,R,VA,DUM1,4,4,0)
      WRITE(6,103)
  103 FORMAT(//////8X,"LATENT",21X,"VECTORS"/8X,"ROOTS"/)
      DO 14 I=1,L
      KK=I*L
      K=KK-L+1
   14 WRITE(6,104) VA(I),(R(J),J=K,KK)
  104 FORMAT(/7X,F8.5,3X,4(2X,F8.5))
      STOP
      END
```

```
C      PROGRAM FOR THE DETERMINATION OF THE UNCORRELATED TIME  SERIES
C
C
C
C
       DIMENSION X(12,80,4),U(960,4),Y(12,80,4),W(960,4),R(4,4),T(4,4),
      1TITLE(20,4,2),EV(4),IYR(80)
       EQUIVALENCE(X(1,1,1),U(1,1)),(Y(1,1,1),W(1,1))
C
C      U OR X  ARE THE PRECIPITATION DATA
C
C      Y OR W  ARE THE UNCORRELATED OR TRANSFORMED DATA
C
       READ(5,99) L,N
    99 FORMAT(2I5)
C      L  IS THE NUMBER OF STATIONS
C      N  IS THE NUMBER OF YEARS OF MONTHLY DATA
C
C      READ THE EIGENVECTORS
       DO 21 I=1,L
    21 READ(5,98) (R(I,J),J=1,L)
    98 FORMAT(4F10.5)
       WRITE(6,400)
   400 FORMAT("1"///5X,"THE DATA FOR THE COMPUTATION ARE"//)
       DO 10 K=1,L
       READ(5,401)(TITLE(I,K,1),I=1,20),(TITLE(I,K,2),I=1,20)
   401 FORMAT(20A4/20A4)
       WRITE(6,402)(TITLE(I,K,1),I=1,20),(TITLE(I,K,2),I=1,20)
   402 FORMAT(//4X,20A4//4X,20A4)
    42 DO 10 I=1,N
    10 READ(5,100) (X(J,I,K),J=1,12),IYR(I)
   100 FORMAT(12F6.2,3X,I4)
       N12=N*12
       CN=N12
       DO 20 J=1,L
       SUMM=0.0
       DO 23 I=1,N12
       W(I,J)=0.0
       DO 22 K=1,L
    22 W(I,J)=W(I,J)+R(J,K)*U(I,K)
    23 SUMM=SUMM+W(I,J)
       EV(J)=SUMM/CN
       A=ABS(EV(J))
       IF(A.LT.0.00001) EV(J)=0.0
    20 CONTINUE
       WRITE(6,201)
   201 FORMAT("1"///28X,"FOR THE TRANSFORMED SERIES"///18X,"DISPERSION MA
      1TRIX",22X,"MEAN VALUES"/)
       DO 11 K=1,L
       DO 13 M=K,L
       SUM=0.0
       DO 12 I=1,N12
```

```
   12 SUM=SUM+W(I,K)*W(I,M)
      T(K,M)=SUM/CN
      A=ABS(T(K,M))
      IF(A.LT.0.00001) T(K,M)=0.0
   13 T(M,K)=T(K,M)
   11 WRITE(6,102) (T(K,M),M=1,L),EV(K)
  102 FORMAT(/8X,4(F8.5,2X),10X,F8.5)
C
        DO 30 K=1,L
      WRITE(6,200) K
  200 FORMAT(*1*//8X,*TRANSFORMED SERIES NO.*,I2/)
      WRITE(6,337)
  337 FORMAT(/1X,4HYEAR,3X,3HJAN,4X,3HFEB,4X,3HMAR,4X,3HAPR,4X,3HMAY,4X,
     13HJUN,4X,3HJUL,4X,3HAUG,4X,3HSEP,4X,3HOCT,4X,3HNOV,4X,3HDEC)
      PUNCH 200,K
      PUNCH 354,(TITLE(I,K,2),I=1,20)
  354 FORMAT(20A4)
      DO 30 I=1,N
      WRITE(6,339) IYR(I),(Y(J,I,K),J=1,12)
  339 FORMAT(/1X,I4,12F7.2)
   30 PUNCH 356, (Y(J,I,K),J=1,12),IYR(I)
  356 FORMAT(12F6.2,3X,I4)
      STOP
      END
```