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SOME DATA ON DIFFUSION AND TURBULENCE  
IN RELATION TO REAERATION

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## ABSTRACT

### SOME DATA ON DIFFUSION AND TURBULENCE IN RELATION TO REAERATION

Heat was used as a tracer to determine diffusion rates immediately below the free surface both in the "film" region and in the remainder of the water in a mixing vessel. The results tend to indicate that a diffusion model can be used to represent the downward transport of a substance which is being absorbed at the free surface. Apparently, the diffusion coefficient in the "film" can be either equal to or greater than the molecular coefficient depending on the amount of mixing at the free surface. Hot film anemometry was used to determine turbulence characteristics from 1 in. to 0.006 in. below the free surface of a laboratory open channel flow. The energy spectra indicate no significant changes in the turbulence in this region. Thus, apparently turbulence exists right up to the free surface and in the "film" region. The amount of reliable data that was obtained was very limited.

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## 1) INTRODUCTION

Domestic and industrial wastes are commonly discharged into rivers and estuaries. A major problem is the determination of the amount and concentration of waste materials which can be assimilated by a given river. This depends to a large extent on the amount of dissolved oxygen which is present in the waterway. In order to determine how much oxygen will be present under a given set of conditions, it is necessary to determine the various ways in which oxygen is supplied and utilized. Furthermore, it is necessary to know the rates at which the processes take place and the factors which influence these rates. One significant source of dissolved oxygen in a river is surface reaeration, i.e. the absorption of oxygen from the atmosphere.

Under a given set of conditions, there exists a concentration of dissolved oxygen for which the solution of oxygen in water will be in equilibrium with the atmosphere to which the water is exposed. This condition is called saturation and the corresponding concentration is given the symbol  $c_s$ . If this equilibrium condition is disturbed, the concentration ( $C$ ) tends to return to the equilibrium condition. For example, if river water is saturated with oxygen and then substances possessing biochemical oxygen demand (BOD) are introduced into the river, the BOD will react with the dissolved oxygen, causing a reduction in the oxygen content. Then, when  $C$  is less than  $c_s$ , oxygen is absorbed from the atmosphere. The rate at which the absorption takes place depends on many factors among which are temperature, type and distribution of oxygen demanding substances, the presence of surface-active agents, and the turbulence level in the water. This report deals primarily with the

relation between reaeration and the turbulence level and related turbulence parameters.

In turbulent water, the rate of absorption of oxygen is assumed to be proportional to the oxygen deficit  $D$  so that (23)\*

$$\frac{dD}{dt} = -K_2 D \quad 1-1$$

where  $D = c_s - C$  and  $K_2$  may be called a reaeration coefficient. If the initial deficit at  $t = 0$  is  $D_0$ , then Eq. 1-1 may be integrated to give

$$\frac{D}{D_0} = e^{-K_2 t} \quad 1-2$$

or

$$\frac{D}{D_0} = 10^{-k_2 t} \quad 1-3$$

where  $k_2 = K_2/2.30$ .

For many years, attempts have been made to find consistently valid correlations between reaeration coefficients such as  $k_2$  and hydraulic parameters. Some of the correlations that have been obtained are reviewed in the next section of this report. As yet, there appears to be no relationship which has received wide acceptance among the various persons dealing with reaeration problems. The author feels that this lack of a generally valid relationship results from the fact that

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\*Numbers in parenthesis refer to references listed at the end of the report.

the majority of the correlations have been obtained primarily by data-fitting techniques. Results from this type of approach will always reflect the particular characteristics of the situation in which they were evaluated. This is true even though attempts are made to include the effects of turbulence. Probably, a generally applicable expression for reaeration coefficients will not result until this expression can be based on an understanding of the mechanism by which turbulence affects the rate of oxygen absorption rather than being based on purely empirical correlations. It is hoped that the research described in this report is one step toward achieving an understanding of the mechanisms involved in reaeration and the relation of these mechanisms to the hydraulics of the flow.

However, until the reaeration mechanisms are fully understood, it is still necessary to analyze pollution problems in rivers. Thus, additional empirical correlations for reaeration coefficients are still appearing in the literature and are finding use in relating reaeration coefficients to mean hydraulic parameters of rivers and estuaries.

## 2) DISCUSSION OF PREVIOUS WORK

### 2.1) General Empirical Approaches

The work of Streeter and Phelps (23) in 1925 is generally considered to have marked the beginning of a new era in the analysis of stream pollution problems. Prior to their work on the Ohio River, the allowable pollution load for a given stream was determined primarily by a "rule of thumb" dilution ratio between sewage discharge and stream flow. Streeter and Phelps took a more rational approach and presented an expression which effectively represented the oxygen balance of a stream under a given set of assumptions. Streeter and Phelps used the equation

$$\frac{dD}{dt} = K_1 L - K_2 D \quad 2-1$$

where  $D$  is the oxygen deficit ( $c_s - C$ ),  $C$  is the average concentration of oxygen at a section,  $c_s$  is the saturation concentration,  $K_1$  is the deoxygenation rate,  $L$  is the amount of oxygen demand in the water,  $K_2$  is the reaeration rate, and  $t$  is time. Eq. 2-1 represents the rate of change of oxygen deficit due to utilization of oxygen (i.e. deoxygenation) by the pollutant and due to surface aeration, which was assumed to be the only source for additional dissolved oxygen. This expression was taken as applying to a one-dimensional element of water as it moved down the river, and  $t$  was interpreted as the time of flow ( $x/U$ ). Thus, Eq. 2-1 was effectively assumed to be

$$U \frac{dD}{dx} = K_1 L - K_2 D \quad 2-2$$

where  $U$  is the mean velocity (which may vary with  $x$ ) and  $x$  is the longitudinal coordinate along the stream.

Integration of Eq. 2-2 leads to the oxygen-sag curve.

Streeter and Phelps made a large number of measurements of oxygen concentration to describe this oxygen-sag curve in various reaches of the Ohio River at various times during the year. By taking values of  $K_1$  determined in the laboratory, they were able to calculate reaeration rates from their data. It was argued that  $K_2$  should be inversely proportional to the square of the depth of the river and directly proportional to the turbulence level of the river, from which it was postulated that

$$K_2 = \frac{mU^n}{H^2} \quad 2-3$$

where  $m$  and  $n$  were assumed to be constants for various stretches of the river and  $H$  was taken as the depth above extreme low water. (Note that  $m$  is not dimensionless.) Values of  $m$  and  $n$  were empirically determined for the various reaches of the Ohio River. Thus, any inaccuracies present in the assumption that  $K_2$  is inversely proportional to  $H^2$  simply influenced the empirical value of  $m$  and  $n$  rather than rendering Eq. 2-3 invalid. In evaluating  $K_2$ , Streeter and Phelps assumed that surface reaeration was the only source of oxygen supply.

The work done by Churchill et al at TVA (2) is another empirical approach. Field measurements were made in rivers which were said to



have essentially no oxygen demand and no source of oxygen other than surface absorption. The river reaches were selected so as to be relatively uniform. From a total of 30 experiments in 5 rivers, it was concluded that  $k_2$  at 20°C could be adequately represented by

$$k_2 = \frac{5U}{h^{5/3}} \quad 2-4$$

where  $k_2 = K_2/2.30$  ( $\text{day}^{-1}$ ),  $U$  is the average flow velocity (ft/sec), and  $h$  is the hydraulic mean depth (ft). In a discussion of the work of Churchill et al, Hull and de Filippi (12) pointed out that Eq. 2-4 was compared only with the data used to evaluate the empirical constants and that no independent verification of Eq. 2-4 had yet been presented.

Changes in turbulence conditions in a river, rather than changes in mean flow parameters per se, account for changes in the re-aeration rate  $k_2$ . Eq. 2-4 gives a correlation between  $k_2$  and two mean flow parameters  $U$  and  $h$ , not between  $k_2$  and turbulence parameters. If there were a unique relation between turbulence conditions and  $U$  and  $h$ , then Eq. 2-4 would effectively relate  $k_2$  to the turbulence conditions, but this is not necessarily the case. Churchill et al point out that either the channel slope ( $S$ ) or the Darcy-Weisbach friction factor ( $f$ ) is needed in addition to  $U$  and  $h$  in order to specify the mean turbulence condition, but they go on to point out that in their work  $S$  was highly correlated with both  $U$  and  $h$ . This seems to be equivalent to saying that all the streams had essentially the same hydraulic roughness. Thus, no improvement in the correlation between  $k_2$  and flow parameters

was achieved by the inclusion of  $S$  in the correlation. It should be pointed out that all the data used in obtaining Eq. 2-4 came from rivers in one section of the country. In other regions where streams have different characteristics from those used by Churchill et al, a different correlation between  $S$ ,  $U$ , and  $h$  may exist and cause Eq. 2-4 to be inaccurate for other regions.

Krenkel (13) and Krenkel and Orlob (14) performed laboratory experiments on the rate of reaeration in turbulent shear flow. They took the coefficient of longitudinal mass transport (i.e. the coefficient of longitudinal dispersion,  $E$ ) as being characteristic of the general state of turbulent mixing in the flow, and reasoned that this coefficient could be used to represent the effects of turbulence on  $k_2$ . A good correlation between  $k_2$  and  $E$  was found for their data. In a discussion of Krenkel and Orlob's paper, Harleman and Holley (9) pointed out that the rate of longitudinal mass transport (i.e. dispersion) which Krenkel and Orlob used results primarily from a convective mechanism rather than being a direct result of turbulence. Thus, it does not seem logical to take the coefficient of longitudinal dispersion as representative of turbulence effects. Harleman and Holley further indicated that longitudinal dispersion usually plays no role in the oxygen absorption process in rivers. However, because of the characteristics of turbulent shear flow, longitudinal dispersion (a convective process) is highly correlated with the average vertical eddy diffusivity (a turbulent diffusion process). Thus, the apparent correlation found between  $k_2$  and  $E$  can more logically be thought of as a correlation between  $k_2$  and the average vertical eddy diffusivity,

which is indicative of the characteristics of the turbulence in the flow. It has more recently been pointed out by Fischer (7) that longitudinal dispersion may depend strongly on transverse velocity distributions. This is another reason that the dispersion coefficient is probably not a good parameter for correlation with reaeration rate coefficients. Krenkel and Orlob also showed that  $k_2$  for their data was related to the rate of turbulent energy dissipation for the flow.

The most recent empirical approach was reported by Thackston and Krenkel (24). They considered the general dependence of reaeration on turbulent diffusion in order to obtain the hydraulic parameters to which  $k_2$  was correlated. From both laboratory and field data, they obtained the expression

$$k_2 = 0.000125 \left[ 1 + \left( \frac{U}{\sqrt{gh}} \right)^{\frac{1}{2}} \right] \sqrt{\frac{g S}{h}} \quad 2-5$$

where  $g$  is the gravitational acceleration, and they indicated that this equation gave a better overall fit to the data than the other generally used equations. Nevertheless, there was still considerable scatter in the data with respect to a line representing Eq. 2-5 and there was one set of data which consistently gave  $k_2$  values about twice as great as would be indicated by Eq. 2-5.

## 2.2) Film Theory

Film theory represents one of the analytical approaches that has been used to study the absorption of gases into turbulent water.

This concept was presented by Whitman (25) in 1923 and by Lewis and Whitman (16) in 1924. As applied to the absorption of oxygen by water, this theory states that the rate of absorption is controlled by the rate of diffusion through a stagnant water film at the water surface. It is further argued that the dissolved oxygen at the surface is in equilibrium with the atmosphere, and thus the concentration at the interface is the saturation concentration,  $c_s$ . Turbulence is assumed to keep the water well enough mixed below the film so that the concentration is uniform everywhere except in the film. Thus, through the thin film, a steep concentration gradient exists, as shown in Fig. 2-1.

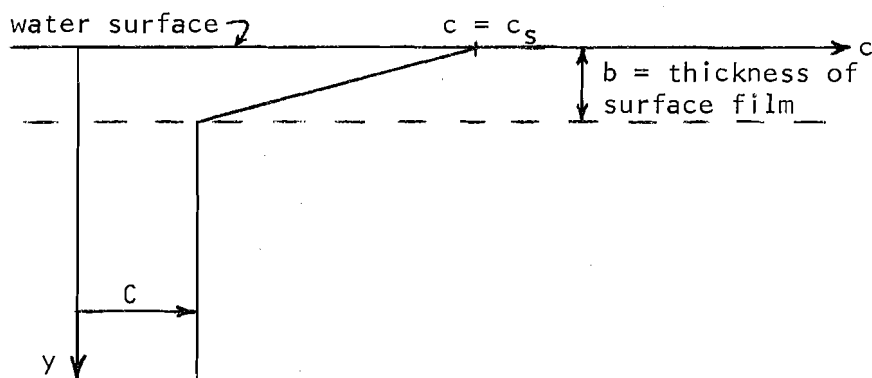


Fig. 2-1: Concentration distribution assumed by film theory.

The diffusion through the film is said to be solely by molecular action since the film is assumed to be stagnant. Thus, diffusion through the film follows Fick's law (1) which states that the rate of mass transport ( $q$ ) through an area ( $A$ ) is equal to a molecular diffusion coefficient ( $D_m$ ) times the negative concentration gradient perpendicular to  $A$ . Applied to a transport in the vertical ( $y$ ) direction, Fick's law is

$$q_y = -A_s D_m \frac{\partial c}{\partial y} \quad 2-6$$

where  $A_s$  is the surface area. The concentration distribution is assumed to be linear through the film so that the film of thickness  $b$ ,

$$\frac{\partial c}{\partial y} = \frac{c_s - C}{b}$$

where  $C$  is the concentration below the film. Fick's law for the film then becomes

$$q_y = -A_s D_m \frac{c_s - C}{b} \quad 2-7$$

The minus sign is sometimes omitted with the understanding that the transport is downward when  $C$  is less than  $c_s$ .

Film theory also led to the definition of a liquid film coefficient ( $K_L$ ) by the expression

$$q_y = A_s K_L (c_s - C) = A_s K_L D \quad 2-8$$

Thus,  $K_L$  is the rate of downward diffusion of oxygen (i.e.  $q_y$ ) per unit surface area ( $A_s$ ) per unit oxygen deficit ( $D$ ). Under the assumptions of film theory,

$$K_L = \frac{D_m}{b} \quad 2-9$$

No method was presented by Lewis and Whitman for finding  $b$  under general conditions. Thus, even if  $D_m$  is known for a given set of conditions and if the assumptions of film theory are valid, this representation is not very helpful for predicting reaeration rates.

### 2.3) Surface Renewal and Related Approaches

Objections were raised to the basic film theory of Lewis and Whitman because it assumed the film to be stagnant. As an alternative, Higbie (10) proposed the penetration theory. This theory for the rate of gas absorption was based on the assumption that the whole body of water was stagnant for short periods of time and, during these periods, oxygen was absorbed and diffused downward solely by molecular diffusion. Then, periodically, the water was instantaneously and completely mixed. This model led to an equation similar to Eq. 2-8 with  $K_L$  given by

$$K_L = 2 \sqrt{\frac{D_m}{\pi t'}} \quad 2-10$$

where  $t'$  is the average time between the complete mixings.

Danckwerts (3) extended Higbie's approach by assuming that various vertical elements of the water could individually undergo complete vertical mixing with different periods between mixings. He assumed that the statistical distribution of the mixing was described by

$$f(t) = re^{-rt} \quad 2-11$$

where  $f(t)$  is the proportional part of the vertical elements for which the elapsed time since the last mixing is between  $t$  and  $t + dt$ . The constant  $r$  may be interpreted as the average rate at which vertical mixing takes place. Thus,  $r$  is analogous to  $1/t'$  of Eq. 2-10.

Danckwerts' approach again led to Eq. 2-7 for the rate of absorption where  $K_L$  was given by

$$K_L = \sqrt{D_m r} \quad 2-12$$

Dobbins (4, 5) used the concept of a surface film, but postulated that the water in the film was periodically mixed with water from below the film. In a sense, this was a combination of film theory and Danckwerts' work. In Dobbins' model, the region below the film was assumed to be uniformly mixed at all times while new surface film was continually being created and then removed. It was assumed that only molecular diffusion takes place while a given parcel of water is in the film. Dobbins also assumed that various parts of the film were mixed or "renewed" at a rate such that Eq. 2-11 described the distribution of times (or ages) since the last renewal within the film. For a film of thickness  $b$ , he showed (4) that these assumptions lead to

$$K_L = \sqrt{D_m r} \coth \sqrt{\frac{rb^2}{D_m}} \quad 2-13$$

As  $r \rightarrow 0$ ,  $K_L$  of Eq. 2-13 approaches  $K_L$  of Eq. 2-9, while for large  $r$ , Eq. 2-13 approaches Eq. 2-12.

In ref. 5, Dobbins describes some experiments which were conducted to investigate the validity of Eq. 2-13. Tests were made on the rates of absorption of helium and nitrogen into water which was being mixed by a vertically oscillating stack of metal grids. By measuring values of  $K_L$  for the two gases, it was possible to calculate values of  $r$  and  $b$ . A constant correlation was found which related the values of  $r$  and  $b$  to the parameters describing the oscillation of the metal grids. As would be expected,  $r$  increased and  $b$  decreased as the speed of oscillation ( $\sigma$ ) of the grids was increased. However, the changes which were found for  $r$  are somewhat difficult to understand. Recall that  $r$  was assumed to be the average rate at which the surface film is renewed. For a  $\sigma$  of 18 rpm (113 rad/min),  $r$  was calculated to be 0.171 per min, while for  $\sigma = 196$  rpm (1230 rad/min),  $r$  was 3780 per min for the same amplitude of screen motion. It is difficult to conceive that increasing  $\sigma$  by a factor of about 11 would have changed the characteristics of the mixing to such an extent that  $r$  would be increased by a factor greater than 20,000.

O'Connor and Dobbins (20) applied Eq. 2-13 to reaeration in natural streams. From typical values for  $r$ ,  $b$ , and  $D_m$ , they argued that the "coth" term in Eq. 2-13 should be essentially unity for natural streams. Then,  $K_L$  would be

$$K_L = \sqrt{D_m r}$$

2-14

This is identical to Danckwerts' expression (Eq. 2-12). Streams were divided into those possessing "nonisotropic" turbulence and those which



were assumed to possess "isotropic" turbulence. The distribution between the types of turbulence was based on the Chezy coefficient  $C_c$ . If  $C_c$  was less than 17, nonisotropy was assumed, while  $C_c$  greater than 17 was taken as implying isotropy. By using this relative low critical value of  $C_c$ , the majority of the streams which they analyzed were classified as having isotropic turbulence. Note, however, that the concept of isotropic turbulence in shear flow (such as a river) is contrary to the basic mechanics of turbulence (21, Chap. 6).

O'Connor and Dobbins argued that  $r$  should be given by the absolute value of the average turbulent velocity fluctuation divided by the turbulent mixing length, where both quantities are evaluated at the free surface. This led to the conclusion, for nonisotropic turbulence, that  $r$  equals the velocity gradient at the free surface. Using the logarithmic velocity distribution to evaluate this gradient for two dimensional flow, they obtained

$$r = \frac{\sqrt{hgS}}{\kappa h} \quad 2-15$$

and

$$k_2 = 480 \left( \frac{D^2 S}{h^5} \right)^{1/4} \quad 2-16$$

where  $\kappa$  = von Karman's constant (taken as 0.4),  $k_2$  = reaeration coefficient ( $\text{day}^{-1}$ ), and ft-sec units are used on the right hand side of Eq. 2-16. (Note that the logarithmic velocity distribution has been found to give poor agreement with the velocities near the free

surface. See ref. 21, p. 489 and 518.) For isotropic turbulence (i.e.  $C_c$  greater than 17),  $r$  was said to be

$$r = \frac{U}{h} \quad 2-17$$

leading to

$$k_2 = \frac{1}{2.31} \sqrt{\frac{D U}{m h^3}} \quad 2-18$$

where  $U$  is the average velocity (ft/sec) and the other terms are defined above. O'Connor and Dobbins presented values of  $k_2$  from field experiments which were in good agreement with their predicted values.

As part of the work presented in ref. 2,  $k_2$  as predicted by O'Connor and Dobbins' theory was compared with values found for some other rivers. From this comparison, it was concluded that there was some question about the validity of using a Chezy coefficient of 17 to distinguish between the two assumed types of turbulence. Also, it was stated that the agreement which was found to exist between the calculated and the observed values of  $k_2$  could have been considerably better. A similar conclusion was reached by Dobbins himself (6). Also, O'Connor (19) suggested using Eq. 2-18 regardless of the value of  $C_c$ .

Dobbins (6) extended the use of Eq. 2-13 to predicting the reaeration rates for rivers and presented a general discussion to show how  $r$  and  $b$  should be related to the fluid properties and the mean flow parameters. Three unknown coefficients were introduced. Estimates

were made for the value of two of these coefficients, and the third one was evaluated empirically on the basis of these estimated values. However, the extreme scatter of the data in the evaluation of this constant did not inspire confidence in the analysis presented.

#### 2.4) Comments on Reaeration Coefficients

The parameters  $K_L$  and  $K_2$  are called reaeration coefficients, or more precisely, reaeration rate coefficients. One might tend to think of them as being dependent on the absorption mechanism at the free surface. However, this is not actually the case. Consider that the water surface is generally assumed to be saturated with dissolved oxygen. This seems to be a reasonable assumption since, at the maximum absorption rates, only about 1 out of every  $10^8$  molecules of oxygen which hits the surface goes into solution (18). If the surface is saturated, the actual absorption process itself can not be the limiting factor in determining how fast the dissolved oxygen content of the water increases. Net absorption of oxygen can take place only as fast as the dissolved oxygen is removed from the surface. Thus, the reaeration rate effectively depends on the downward rate of transport of dissolved oxygen at the surface. This rate of transport, depends in turn, on the turbulence conditions near the free surface. In some of the literature reviewed above, the concept of surface renewal was used to represent the influence of turbulence on this transport rate. The data presented in this report tends to indicate that a diffusion model may also be used to represent the transport rate.

If the diffusion model is valid, one may write by analogy to Fick's Law (Eq. 2-5)

$$q_y = -A_s e \frac{\partial c}{\partial y} \quad 2-19$$

where  $e$  is the diffusion coefficient at the surface and may differ from  $D_m$ , in general. Further, if the film is defined as the region in which a steep concentration gradient exists immediately below the free surface and if there is a linear gradient in this film, then

$$K_L = \frac{e}{b} \quad 2-20$$

if Eq. 2-8 is taken as a general definition for  $K_L$ . This equation gives  $K_L$  as the ratio between the diffusion coefficient at the surface and the thickness of the hypothetical surface film. The film thickness is used only as a means for defining the concentration gradient at the free surface. Both  $e$  and  $b$  no doubt are related to the turbulence conditions near the free surface. Eq. 2-20 is presented, not because it offers an immediate means for predicting  $K_L$  values but rather because it may offer a means for eventually relating  $K_L$  to turbulence and to mean hydraulic parameters.

Whether the diffusion model or the surface renewal model is used, it must be realized that the mechanism which controls the rate of reaeration is the downward transport of dissolved oxygen at the free surface, or effectively the turbulence near the free surface. Thus, it seems that  $K_L$  is conceptually a better coefficient to use than  $K_2$ ,

since for a given turbulence condition near the surface  $K_L$  would not change for different depths but  $K_2$  would change. Consider the different significance of  $K_2$  and  $K_L$ . From Eq. 2-8, the transport rate is

$$q_y = -A_s K_L D \quad 2-21$$

while from Eq. 1-1, considering the relation between  $dD/dt$  and  $q_y$ , one may also write

$$q_y = -VK_2 D \quad 2-22$$

Thus,  $K_L$  represents the transport (or absorption) rate per unit surface area while  $k_2$  is a rate per unit volume ( $V$ ). If the mean depth  $h$  is defined by  $h = V/A_s$ , then

$$K_L = K_2 h \quad 2-23$$

so that in seeking mathematical correlations it makes little difference whether  $K_L$  or  $K_2$  is used as long as  $h$  is considered as one of the parameters on which the reaeration coefficient depends.

### 3) DIFFUSION MEASUREMENTS

As pointed out in Eq. 2-20 and the accompanying discussion, it may be possible to represent the downward transport of oxygen away from the free surface by a diffusion model and to relate the reaeration coefficient  $K_L$  to the diffusion coefficient in the surface film which is defined for present purposes as the region of the steep concentration gradient immediately below the free surface.

#### 3.1) Diffusion Equation

For the case of a conservative substance which is absorbed at the free surface turbulent water and diffused downward in the absence of any net velocities, the equation for the conservation of that substance may be written (1, p. 627-9) as

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial y} \left( e_y \frac{\partial c}{\partial y} \right) \quad 3-1$$

where  $c$  is the concentration of the substance (turbulent fluctuations having been averaged out),  $y$  is the vertical coordinate,  $e_y$  is the vertical diffusivity for the substance under consideration and may be a function of  $y$ , and  $t$  is time. In general, if  $c$  is measured as a function of  $y$  and  $t$ , this data may be used to calculate the eddy diffusivity and thereby gain some insight into the transport process and the way in which it is affected by the turbulence which is present.

### 3.2) Determination of Diffusion Coefficient

If a steady state concentration distribution can be obtained, then  $\partial c/\partial t = 0$  and the diffusion coefficient can be calculated rather easily. If the substance which is being absorbed across the free surface is either being used-up at the bottom of the water or is diffusing out the bottom at the same rate that absorption is taking place at the surface, then a steady state concentration distribution should result. As described later, these experiments sought to use a steady state concentration distribution. For this situation (i.e.  $\partial c/\partial t = 0$ ), Eq. 3-1 can be integrated once with respect to  $y$  to give

$$e_y \frac{\partial c}{\partial y} = m \quad 3-2$$

where  $m$  is a constant. If  $e_y$  and  $dc/dy$  are known at some  $y$  (again the situation for these experiments),  $m$  can be found. The gradient at other values of  $y$  (i.e.,  $dc/dy$ ) can then be used to evaluate  $e_y$  as a function of  $y$ . The constant  $m$  is proportional to the vertical flux of the diffusing substance. The exact relation between the constant in Eq. 3-2 and the vertical flux depends on the way in which the concentration  $c$  is defined. (See ref. 1, Chap. 20.)

### 3.3) Experimental Program

To the writer's knowledge, there is no sensor which is physically small enough to allow for the measurement of the steep dissolved oxygen gradients which apparently exist in the film region. Since small

temperature sensors are available, it was decided to use heat as a tracer to study the diffusion downward from a free surface. A situation analogous to that for oxygen absorption was created for the laboratory study. In a mixing vessel, the air over the water was heated to a temperature higher than the water temperature. The temperature difference caused heat to be absorbed by the water at the free surface and diffused downward just as with oxygen in the reaeration process. The sides of the vessel were insulated with 12 in. of fiber glass to assure that diffusion of heat took place only in the vertical direction. Temperatures were measured in the water as a function of time and distance below the free surface. The temperature data was used to calculate then diffusion coefficient in accordance with Eq. 3-2.

### 3.3.1) Mixing Vessel

The tank in which the temperature profiles were measured was a 20" x 12" x 12" deep vessel in which stainless steel screens oscillated vertically to produce turbulence. The six screens (flattened expanded metal, 1/2" no. 16) had a vertical spacing of 1" and were driven by four 5/16" rods. (See Fig. 3-1.) The screens were fastened with nuts and washers to the four 5/16" threaded stainless steel rods, which extended through the screen openings. The top screen had a nut and washer only on its lower side. This arrangement allowed the top screen to come close to the water surface without having a nut on top of the screen which would break the water surface. Each stainless



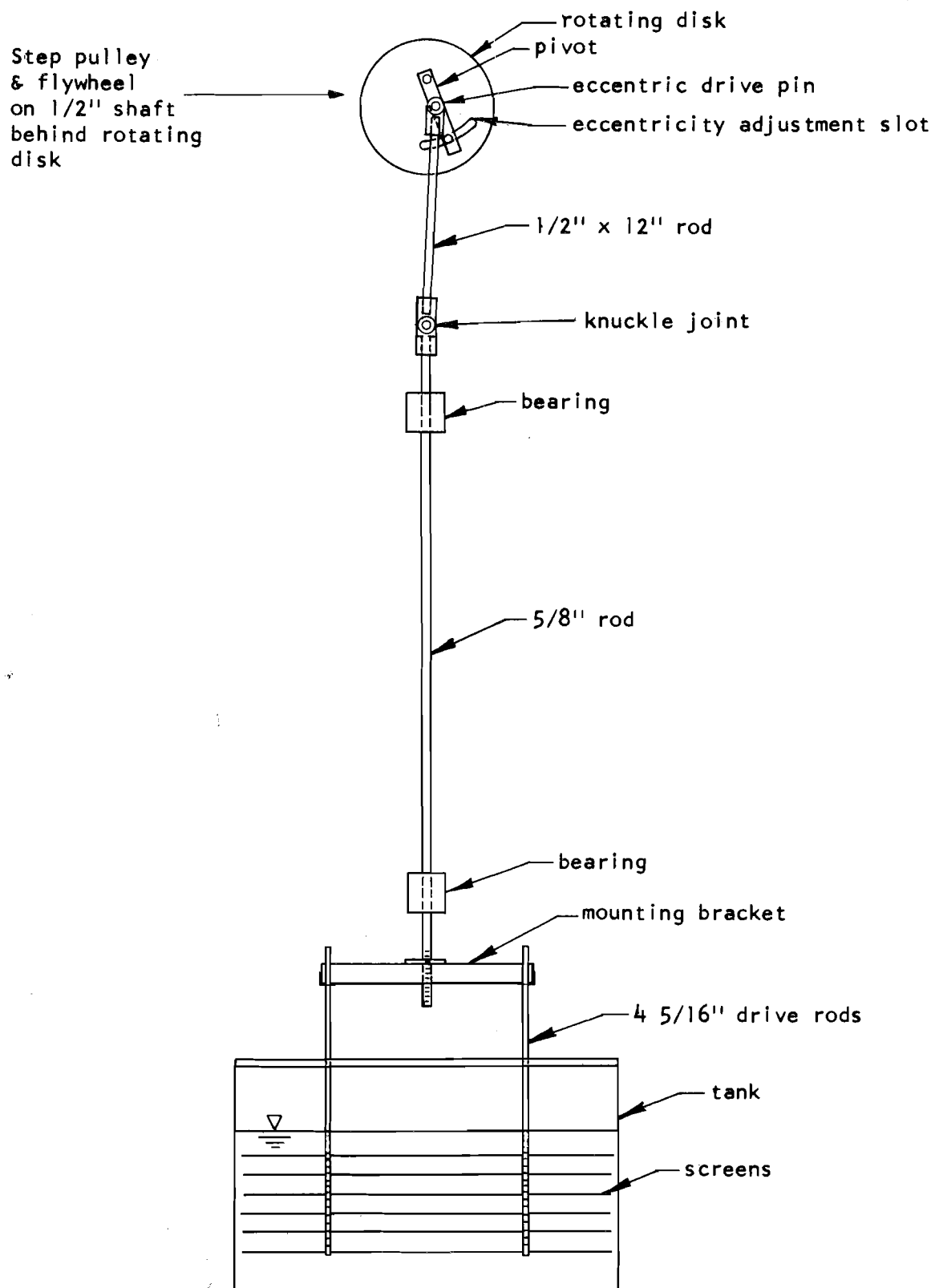


Fig. 3-1: Schematic diagram of mixing vessel

rod was threaded just up to the level of the top screen so that only the smooth rod (no threads) extended through the water surface.

Experiments were run with a 7/16" stroke between vertical extremes of the screen movement and at 100, 150, and 200 rpm. The average distance of the top screen below the water surface could be changed by varying the amount of water in the tank. The air space over the water in the tank was heated by the use of silicone-rubber-embedded heating tapes. With these tapes, the chance of heating by radiation was small because of the relatively low temperature of the tapes. Nevertheless, aluminum foil shielding was used to assure that no heat was transferred to the water by radiation.

### 3.3.2) Thermistor Circuit

The temperature sensor was a 0.014"-dia., glass-coated, bead thermistor (Fenwal Electronics, Framingham, Massachusetts, no. GC32J1). It was mounted at the end of a conical nose on a 1/4" O.D. plexiglass tube.

As shown in Fig. 3-2, the sensor was one leg of a Wheatstone bridge. The bridge was connected to a Sanborn recorder (model 140) for excitation and amplification of the output from the bridge. Voltage across the sensor was restricted to less than 30 millivolts in order to keep the sensor from heating more than 0.01°C above its surroundings. An ammeter ( $\pm$  50 milliamps) was used to indicate when the bridge was balanced. The ammeter circuit had a coarse and fine switch which referred to the sensitivity of the circuit.

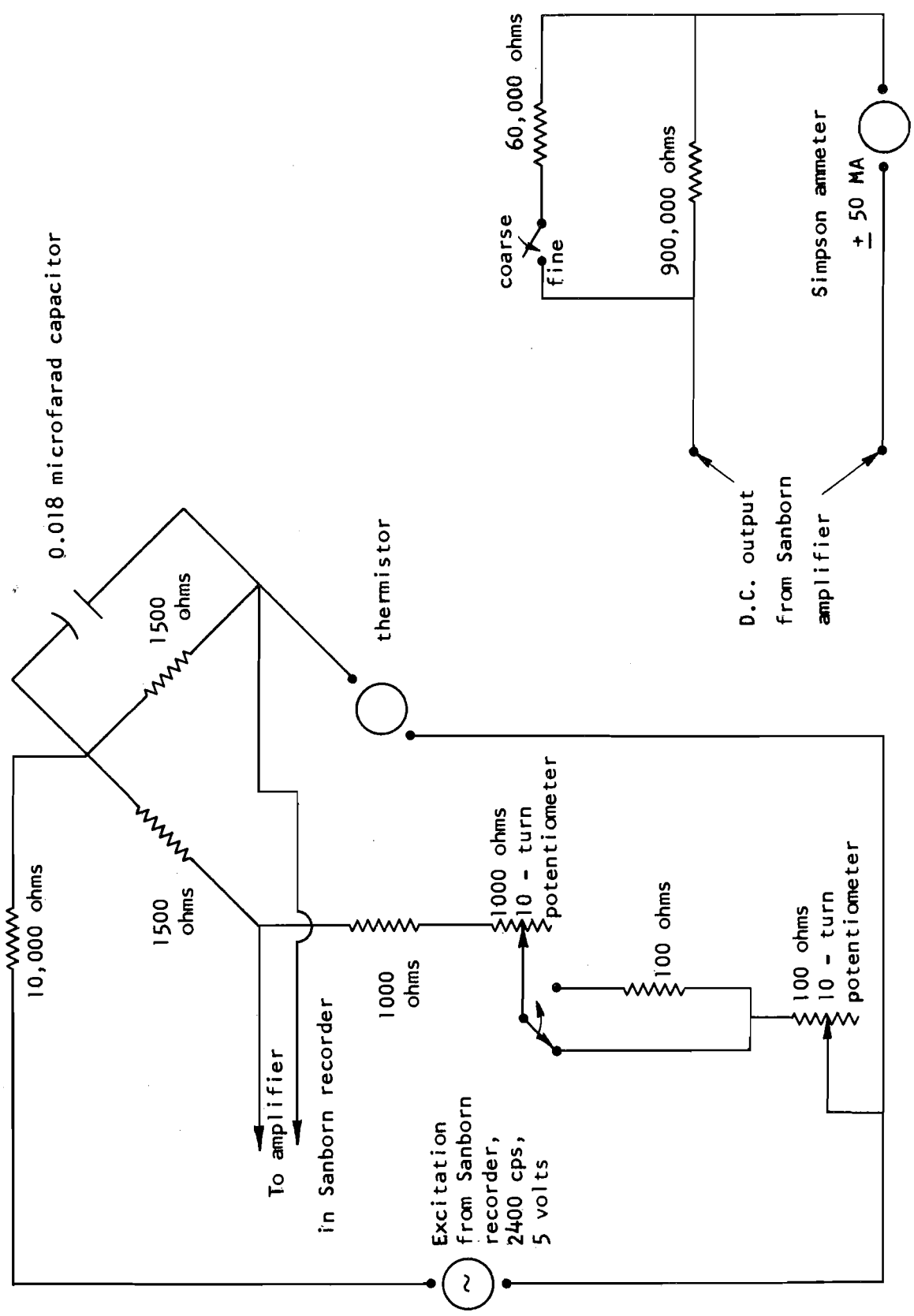


Fig. 3-2: Circuitry for temperature measurement

Usual operation of the temperature measuring device was to balance the bridge with the thermistor at a known temperature using the 1000 ohm potentiometer (Fig. 3-2) and then to lock this potentiometer. Temperature measurement was accomplished using the 100 ohm potentiometer to bring the bridge back to balance as the resistance of the thermistor changed due to temperature changes. The 100 ohm fixed resistor could be added to the circuit to give a total variation of 200 ohms in one leg of the bridge. This allowed temperature measurement over a range of  $3.7^{\circ}\text{C}$  from the balance point.

The thermistor was calibrated against a calorimeter thermometer (ASTM 56c) which was marked directly to  $0.02^{\circ}\text{C}$ . The calibration curve had a slight curvature, but could be considered linear over a range of about  $2^{\circ}\text{C}$ . The scattering of the data on the calibration curve was all within  $\pm 0.05^{\circ}\text{C}$  on the best fit line (Fig. 3-3).

### 3.3.3) Secondary Mechanisms of Heat Transfer to the Water

Since heat was being used as a tracer to measure diffusion rates, it was necessary to investigate possible heat losses from the mixing vessel and possible heat transfer to the water other than across the free surface. For the water  $10^{\circ}\text{C}$  warmer than room temperature, the heat loss through the insulated walls was calculated to give a temperature change of about  $0.04^{\circ}\text{C/hr}$ . Therefore, this mechanism for heat loss was considered negligible, especially since the water temperature normally was much less than  $10^{\circ}\text{C}$  above room temperature. Also, under typical operating conditions, but with the water surface covered to prevent heat transfer, it was empirically determined that heat transfer through

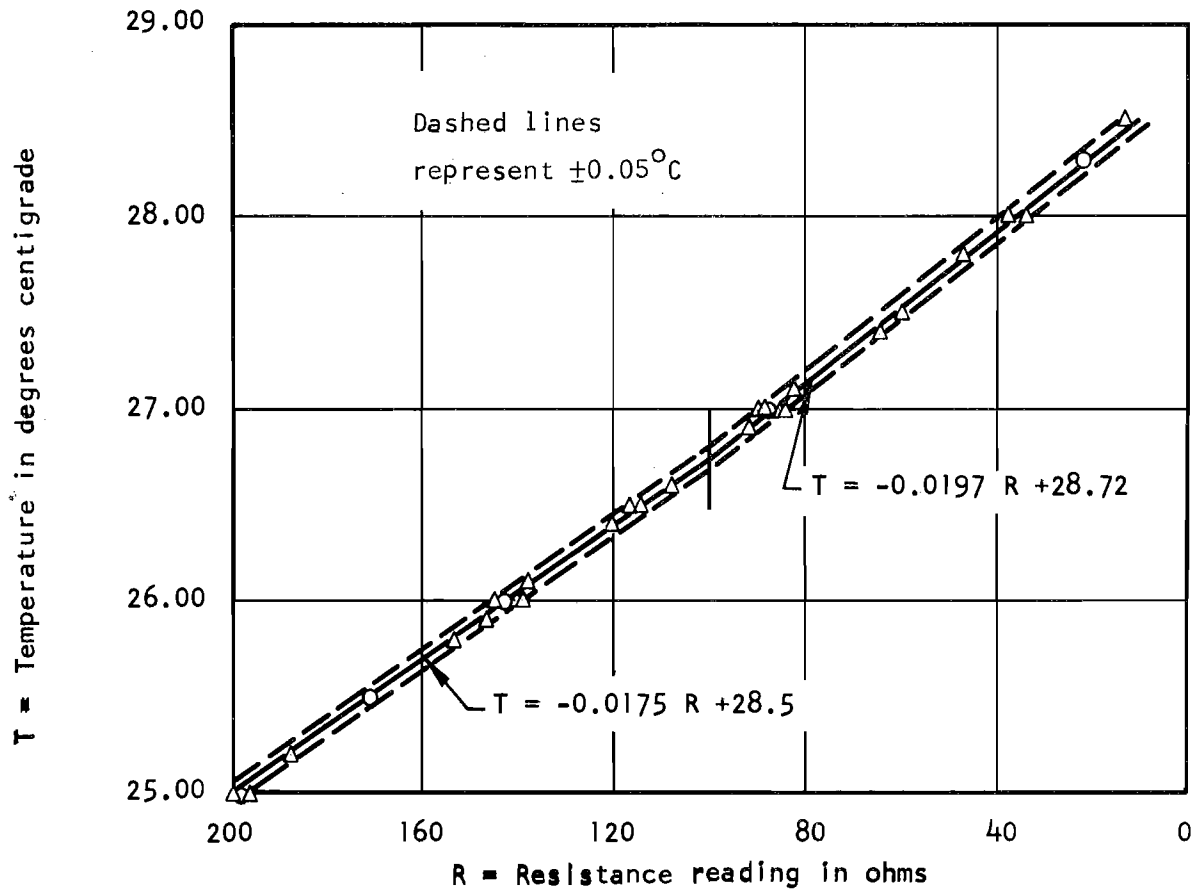


Fig. 3-3: Calibration of thermistor

the tank walls and the steel rods did not cause any significant heating of the water.

#### 3.3.4) Procedure

Water was added to the mixing vessel and brought to the desired temperature by immersion heaters, and the air space was heated. The temperature of the air space in the tank was automatically controlled to within about  $\pm 0.2^{\circ}\text{C}$ . The temperature probe was mounted in a point gauge at the middle of the tank's length and 3-1/4' in from one side. With the screens stopped, the point gage was used to locate the water surface. The screens were then started, and measurements of temperature at various depths and times were then taken. Repeated vertical temperature traverses were made to assure that steady state had been reached.

It was necessary to calibrate the thermistor each day that data was taken, but calibration checks at the beginning and end of each set of data showed that the thermistor gave the accuracy  $\pm 0.05^{\circ}\text{C}$  previously mentioned.

#### 3.3.5) Results

The amount of data that was obtained and that could be considered reliable was quite small, due to many experimental problems. The greatest problem was slow, fluctuating temperature variations near the surface. These variations rendered a large part of the data essentially useless. It is supposed that the oscillating screens set up a slow circulation in the tank in addition to the mixing that would be

expected from the vertical movement of the screens. Extreme care was used in trying to assure that the screens were horizontal and flat and that the drive rods were vertical in order to try to prevent the circulation from developing. In spite of all these precautions, the circulation still seemed to develop in a rather unpredictable fashion. It does not seem that the temperature fluctuations could have been due to seiche-type waves since the period for such waves would have been much shorter than the period of the actual fluctuations (order of 5 sec). Also, the fluctuations were not correlated with the on-off cycle of the heaters which were used to control the air temperature.

Three sets of data that did have an essentially steady temperature distribution are shown in Figs. 3-4, 5, 6, along with conditions under which each set was taken. The values of the diffusion coefficients at the water surface are also shown. These values were calculated by Eq. 3-2, using the fact that in the region occupied by the screens (8)

$$e_y = 0.172 a^2 \sigma \quad 3-3$$

where  $a$  is the amplitude of the oscillation and  $\sigma$  is the frequency.

Although this is certainly very limited data, there are some points worth noting. For the lowest frequency of oscillation (100 rpm, Fig. 3-4) the diffusion coefficient is approximately 1.3 times the molecular coefficient ( $0.0015 \text{ cm}^2/\text{sec}$ , ref. 1). When the frequency was increased to 200 rpm (Fig. 3-5), the diffusion coefficient increased by a factor of about 2.5. This seems to be a much more reasonable change

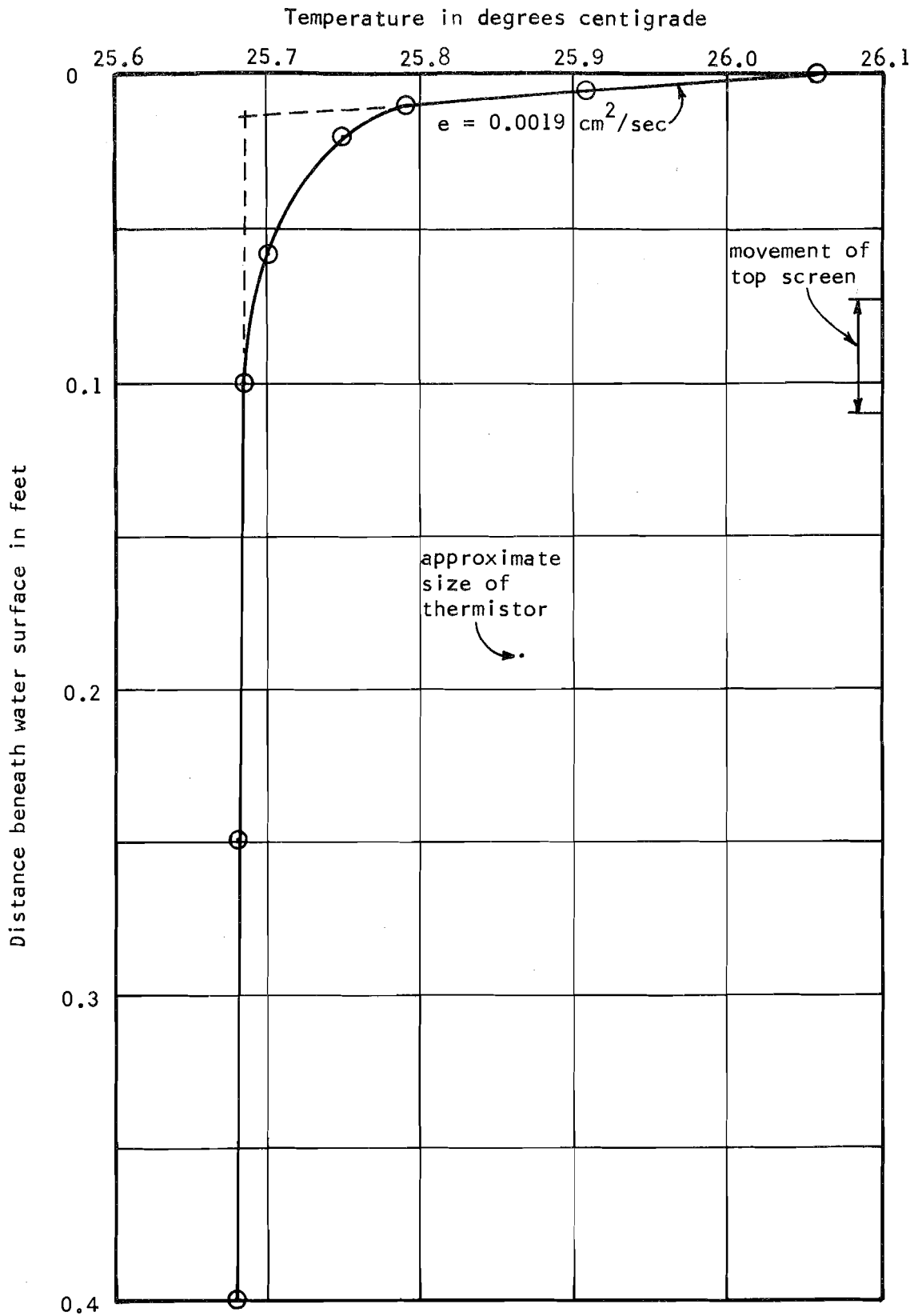


Fig. 3-4: Temperature profile ( $\sigma = 100$  rpm)



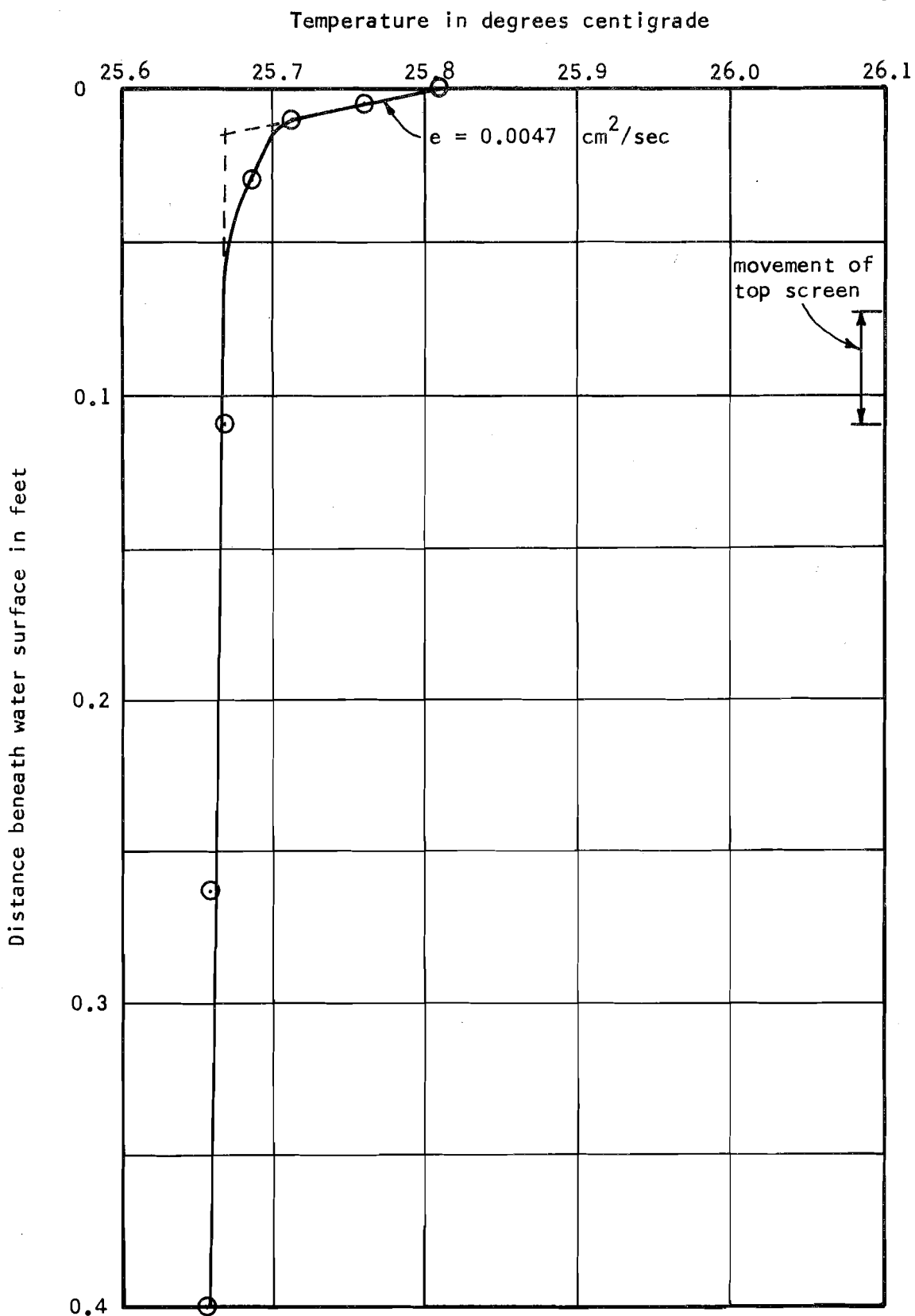


Fig. 3-5: Temperature profile ( $\sigma = 200$  rpm)

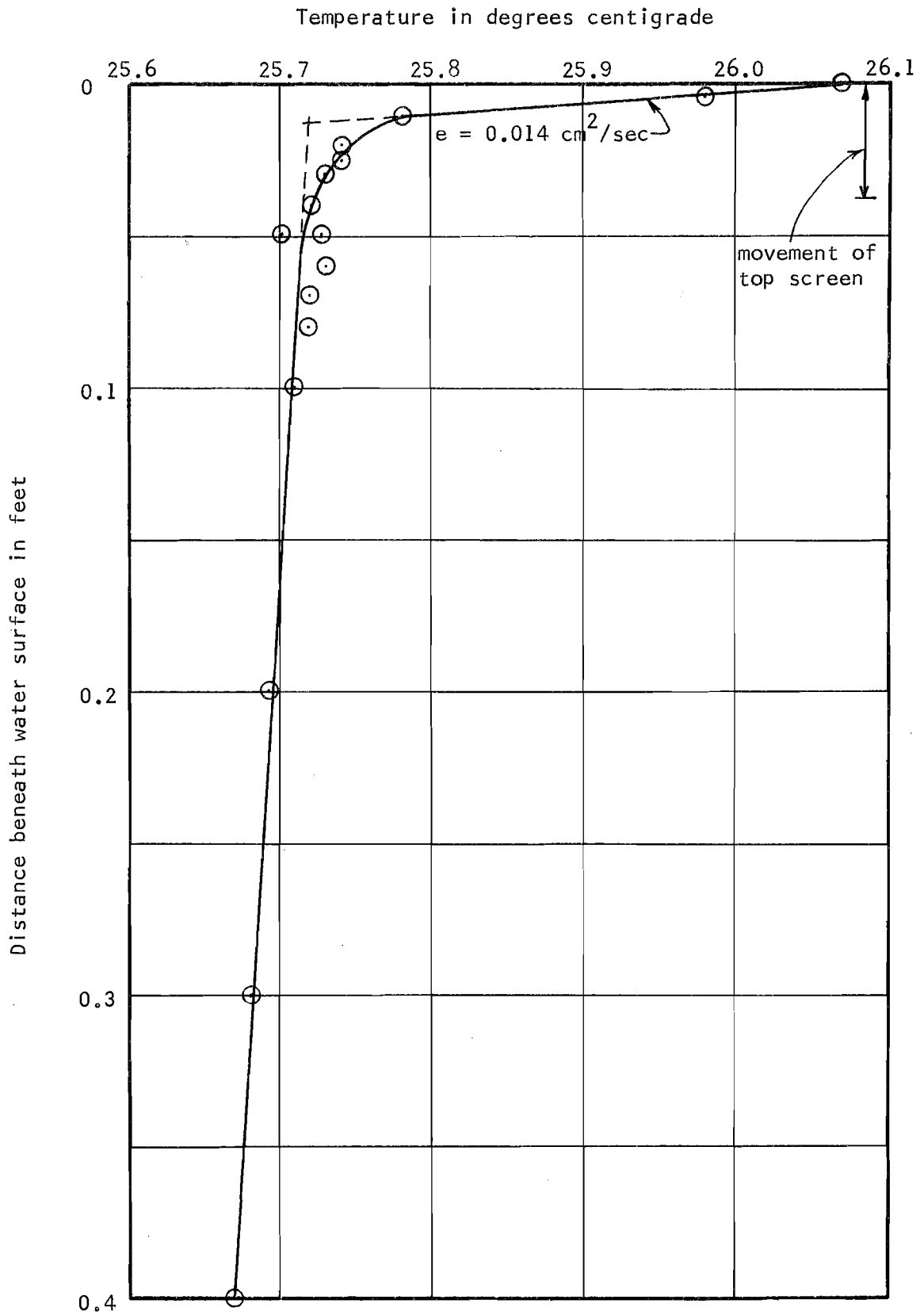


Fig. 3-6: Temperature profile ( $\sigma = 150 \text{ rpm}$ )

than the variation that was found in the surface renewal rate when the frequency was increased. (See Section 2.3.)

Fig. 3-6 shows data taken when the top screen came right up to the water surface but did not break the surface. For this case, the temperature distribution was still linear. There was an increase by a factor of approximately 75 in the diffusion coefficient as compared to Fig. 3-4, but the diffusion coefficient was still about 240 times less than that in the lower regions of the water.

A surface film thickness may be defined by extending the two linear portions of the temperature distribution until they intersect. For the oxygen absorption problem, this film thickness and the diffusion coefficient at the surface would be sufficient to determine the reaeration coefficient  $K_L$  (Eq. 2-20). The fact that these two straight lines do not correctly represent the entire concentration (temperature) distribution does not really matter since only the gradient and the diffusion coefficient at the surface are needed to determine the rate of absorption.

### 3.3.6) Summary

On the basis of very limited data it appears that there may be some validity to the use of a diffusion model to represent the downward transport immediately below the free surface when a substance is being absorbed at the free surface. The diffusion coefficient at the surface apparently can vary upwards from the molecular value as the amount of mixing at the surface increases.

If more extensive tests validate the diffusion model, then it would be hoped that the diffusion coefficient at the surface could be related to the nature of the turbulence. Part of this work would be to seek to explain the mechanism for the diffusion in view of the fact that the diffusion coefficient can apparently lie between the molecular value and the turbulent value, even when there is strong mixing at the surface (Fig. 3-6). The explanation for this may be in the fact that the substance absorbed at the surface moves into the turbulent eddies under molecular action at the same time that the eddies are moving about.

#### 4. TURBULENCE MEASUREMENTS

The measurements described in this section were undertaken in an effort to gain some insight into the turbulence condition in open channel shear flow near the free surface and possibly in the film region itself. Hot-film anemometry was used in this study to measure characteristics of turbulence. The anemometer gives essentially instantaneous, point velocity readings in the form of a continuous output voltage. This continuous voltage signal can be used to obtain turbulence information such as the root-mean-square (RMS) and energy spectrum of the velocity fluctuations.

##### 4.1) Turbulence Characteristics

The primary characteristics of the turbulence on which data was sought were the intensity and the one-dimensional spectrum. The intensity is defined as

$$\frac{\sqrt{\overline{u'^2}}}{\bar{u}}$$

where  $\bar{u}$  is the time-averaged velocity and  $u'$  is the turbulent fluctuation, i.e.  $u' = u - \bar{u}$ , where  $u$  is the total instantaneous velocity. The one-dimensional spectrum  $E(n)$  is defined so that  $E(n)dn$  is proportional to the kinetic energy of the turbulence in the frequency band  $n$  to  $n + dn$ , where  $n$  is frequency; thus, (11),

$$\overline{u'^2} = \int_0^{\infty} E(n) dn$$

The normalized spectrum,  $F(n)$ , is similarly defined by

$$F(n) = \frac{E(n)}{u'^2} \quad 4-2$$

so that

$$\int_0^{\infty} F(n) dn = 1 \quad 4-3$$

Except for the high frequencies, the normalized spectrum should have the form (11, 15, 21)

$$F(n) = \frac{4 \frac{L_x}{u}}{1 + \left[ \frac{2\pi n L_x}{u} \right]^2} \quad 4-4$$

where  $L_x$  is the macroscale.

#### 4.2) Anemometry Equipment and Measurement Techniques

In these experiments, a constant temperature anemometer (Heat Flux System Model 1010 manufactured by Thermo-Systems Incorporated of St. Paul, Minnesota) was used with a Thermo-Systems hot-film sensor having a 0.006" diameter quartz coated platinum-film sensor and a sensitive length of 0.080". Detailed information on the principle of operation and techniques for using a hot-film anemometer may be found in ref. 11 and in the manufacturer's publication. The sensor resistance is related to the operating temperature by the equation

$$\frac{R}{R_e} = 1 + \rho(t_s - t_e)$$

4-5

in which  $R$  is the resistance of the sensor at the operating temperature ( $t_s$ ) and  $R_e$  is the "cold resistance" at the temperature ( $t_e$ ) of the environment. The ratio  $R/R_e$  is called the "overheat ratio." The coefficient  $\rho$  is the temperature coefficient for the film material and is given by the manufacturer as  $0.00234/^\circ\text{C}$  for their platinum films. An overheat ratio of 1.05 was used for all tests. This corresponds to sensor temperature  $21.4^\circ\text{C}$  above the water temperature, e.g. a sensor temperature of  $46.4^\circ\text{C}$  for a water temperature of  $25^\circ\text{C}$ .

The anemometer was calibrated in the flume by using a pre-calibrated current meter to determine the velocity at the sensor. A typical calibration curve is shown in Fig. 4-1. The current meter was a Stevens Midget Current Meter which has a propeller that is about 1-in. in diameter. During calibration, the flow depth was 6 in. and the hot-film sensor and the center of the propeller meter were placed about 1.5" below the free surface so as to be in a region of an essentially flat velocity distribution. A typical velocity distribution on the vertical centerline is shown in Fig. 4-2.

The sensor was mounted on an  $80^\circ$  elbow on the bottom of a vertical traversing rod. This elbow allowed the sensor to be brought up below the free surface without breaking the surface.

The circuitry and instruments used to process the output of the anemometer are shown schematically in Fig. 4-3. As will be discussed in more detail later, the turbulence had significant components at frequencies less than 2.5 Hz. These low frequency components interfered

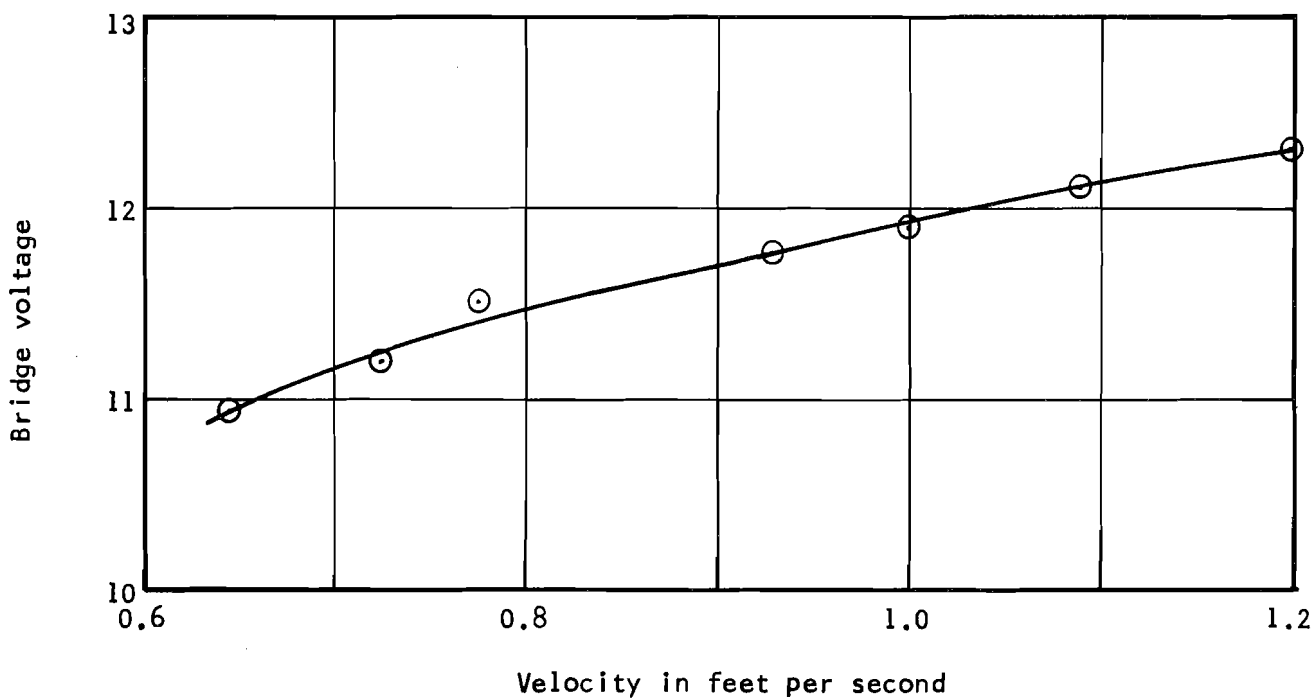
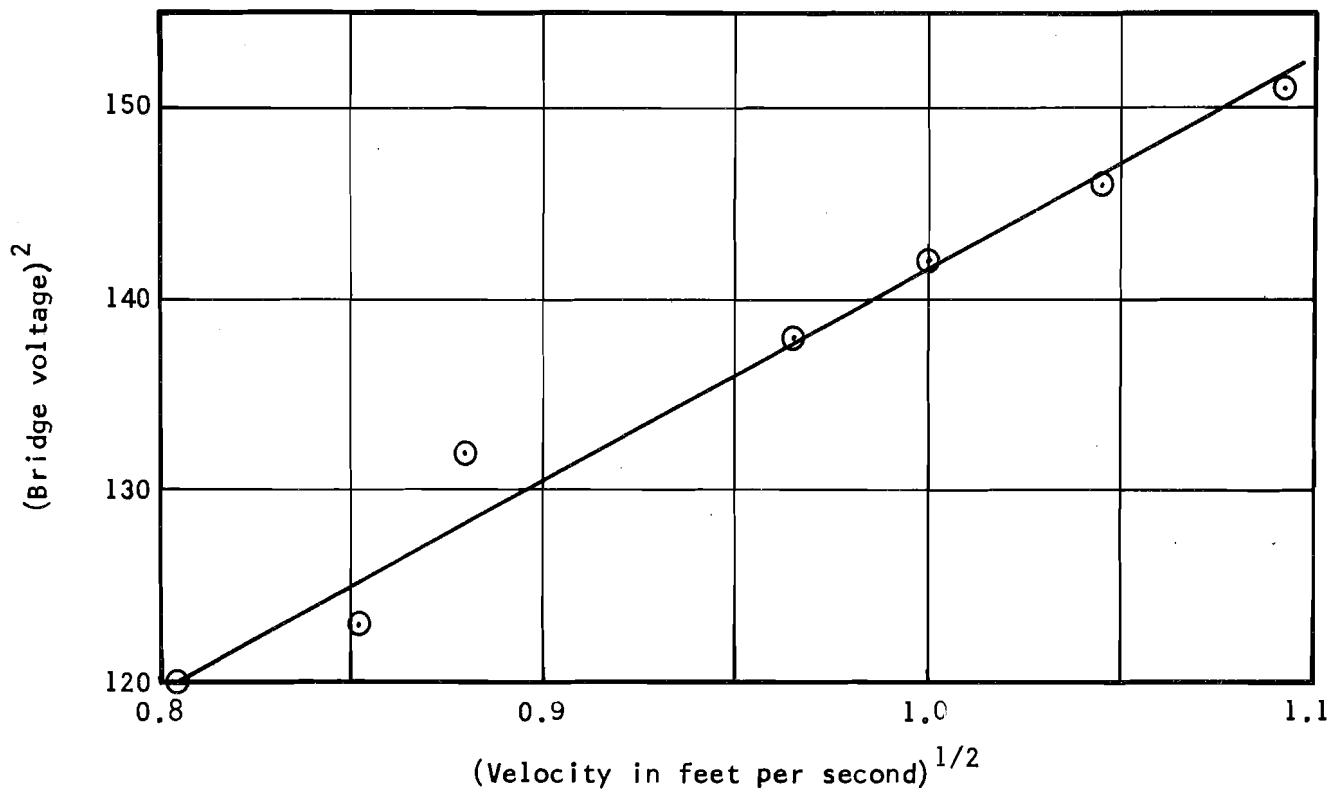


Fig. 4-1: Sample anemometer calibration curve



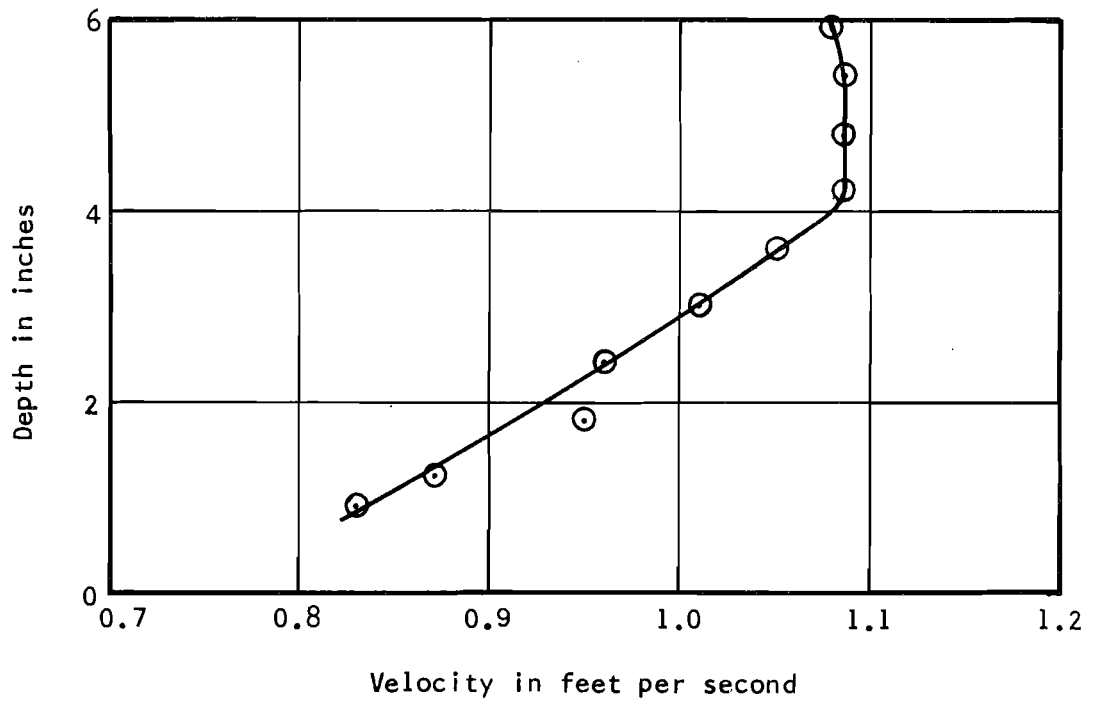


Fig. 4-2: Velocity distribution on vertical centerline at measurement station

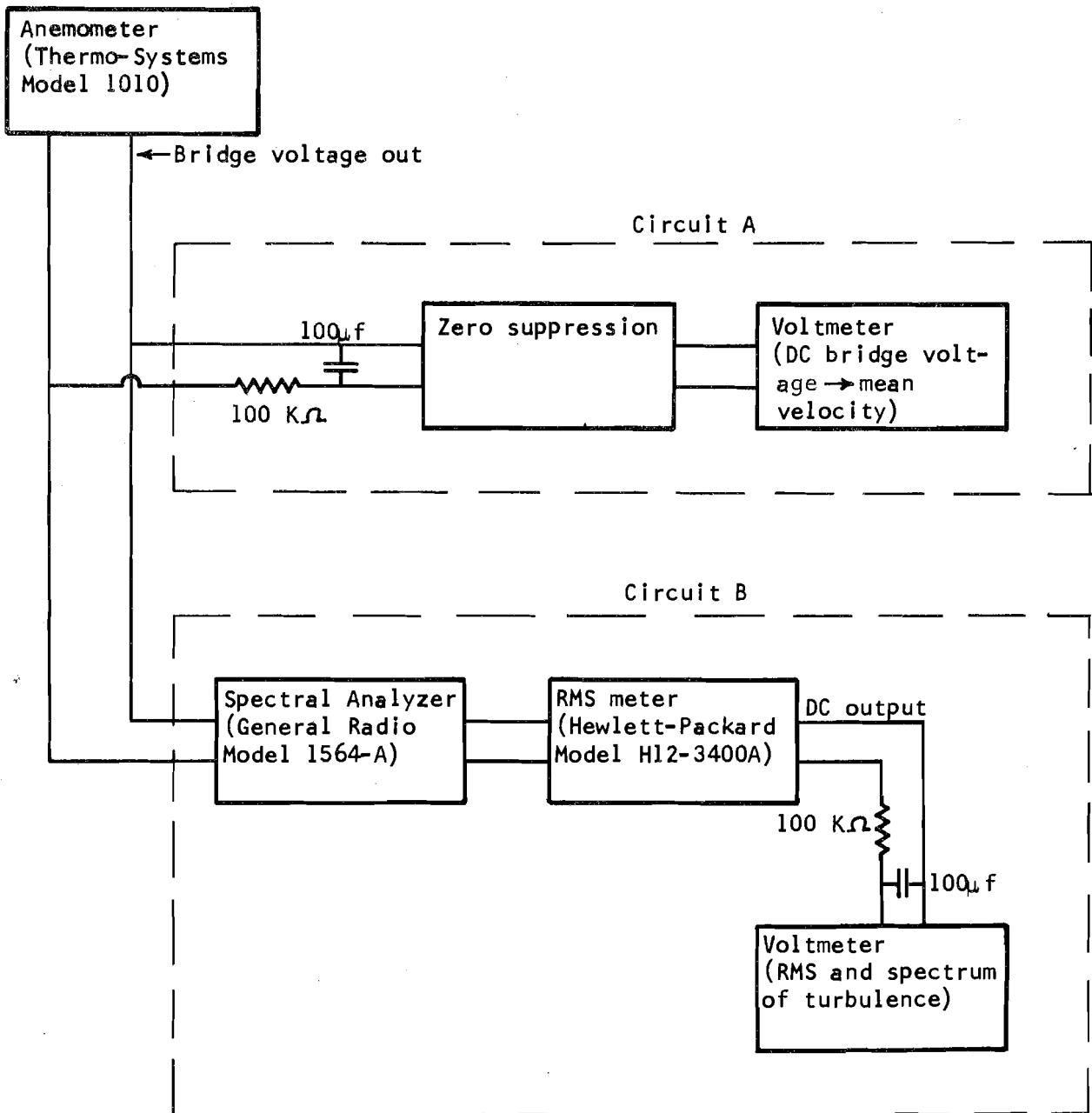


Fig. 4-3: Schematic diagram of data analysis circuits

with attempts to use the anemometer panel-meter for reading the D.C. voltage which is proportional to the mean velocity. Thus, an external circuit (A, Fig. 4-3) was used to filter out the low frequency components. The R-C filter shown has a corner frequency of 0.16 Hz. Circuit A, Fig. 4-3, was used to calibrate the sensor and to measure mean velocities.

The turbulence data was analyzed by use of Circuit B shown in Fig. 4-3. The bridge voltage from the anemometer was fed into the Sound and Vibration Analyzer, which is essentially a tuneable band pass filter with center frequencies variable from 2.5 Hz to 25 KHz. The 1/10-octave (7%) band width was used. This band width is the one given by the manufacturer and is defined by the -3db width. In analyzing the turbulence data, a band width of 15% was used. This width was obtained as the width of a rectangular response curve of unit height which had the same area as the actual response curve. The output of the Sound and Vibration analyzer was that part of the turbulence signal passing through the selected band. This output was fed into a Hewlett-Packard Model H12-3400A RMS voltmeter modified to have a lower frequency response (-4db) of 2Hz. Again, it was found that the turbulence components at frequencies less than 2.5 Hz interfered with reading the RMS voltmeter, so the DC output of the RMS meter was fed through another RC filter to an external meter. The voltage read from Circuit B, Fig. 4-3, was used to obtain the spectral data in accordance with the definitions in Section 4.1.

It should be noted that the use of the circuits shown including the RC filters did not allow measurement of the turbulence components

at frequencies less than 2.5 Hz, it only prevented these components from interfering with the other measurements. Some information about the low frequency components could probably have been obtained with this analog-type analysis by recording the anemometer output voltage on a tape recorder and playing it back at a faster speed for analysis.

#### 4.3) Flume

A closed system recirculating flume was used for the turbulence measurements. A plan-view sketch of the flume is shown in Fig. 4-4, where some of the dimensions are also indicated. The flume had horizontal bed and a smooth (epoxy-painted wood) flow surface. The flow section was 30 ft long by 11 1/4 in. wide by 10 1/2 in. deep with a streamlined entrance composed of 11-in. long quadrants of an ellipse on the two sides and bottom. Just upstream of the entrance there were five layers of window screen (18 by 14 mesh) spaced 3/4" apart. The head and tail boxes were each 22 in. wide by 34 in. long by 36 in. deep (vertically). The water level in the tail box prevented drawdown of the water surface in the flume. The turbulence measurements were made on the centerline of the flume five feet upstream of the tailbox. This location was verified to be far enough upstream of the tailbox so as to avoid the effects of the separation due to flow into the tailbox. Also, it was verified to be far enough downstream of the entrance so that the boundary layer was fully developed.

In order to prevent air bubbles from forming on the sensor due to the heated film driving dissolved gases out of solution from the water, the water was deaerated by a method previously described (17). Also,

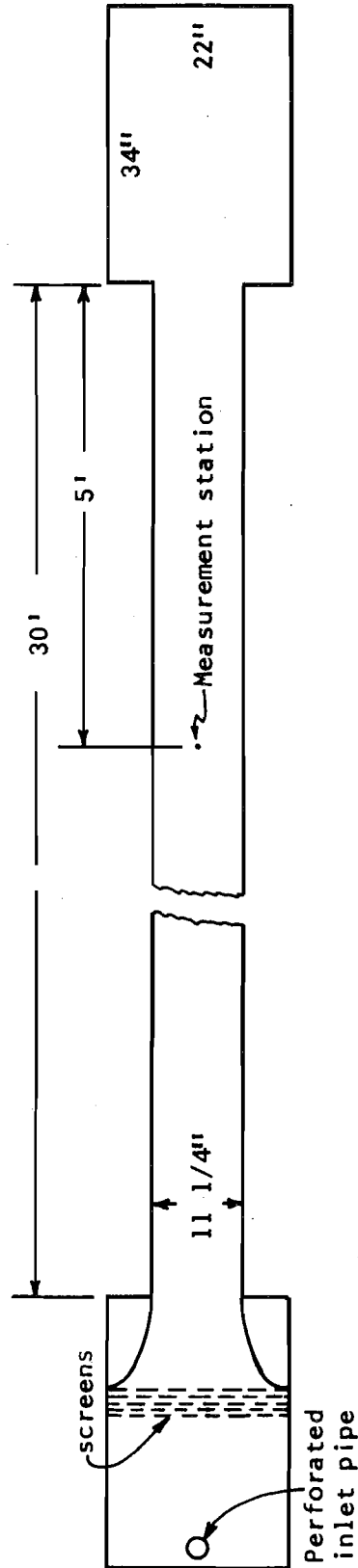


Fig. 4-4: Schematic plan view of flume

the water in the flume system was continuously filtered through a diatomaceous earth filter (Sear Model 167.4373 filter, with 15 sq ft of filter area, for portable swimming pools). These methods proved sufficient to keep the hot film free of both gas bubbles and dirt.

#### 4.4) Results

Data was taken for only one flow condition, namely a depth of 6" and a mean velocity of 0.93 fps. The corresponding Reynold number was 104,000 based on a length scale of 4 times the hydraulic radius.

The turbulence spectra (E) for frequencies of 2.5 Hz and greater are shown in Fig. 4-5 for distances of 1.0 in. to 0.006 in. below the free surface. It should be noted that the diameter of the hot film was 0.006 in. and that the film caused some displacement of the free surface when it was placed 0.006 in. and 0.01 in. below the free surface. Also, the accuracy of the placement of the film at these two distances below the surface is certainly open to question because the surface itself had slight ripples on it.

It is estimated that a lower bound for the thickness of the film for oxygen absorption for this flow would be on the order of 0.001 in. This value was obtained by use of Eqs. 2-20 and 2-23 with  $e$  taken as  $2 \times 10^{-5} \text{ cm}^2/\text{sec}$  (an approximate value for the molecular diffusivity) and with  $k_2$  estimated as  $3 \times 10^{-5} \text{ sec}^{-1}$  from Eq. 2-5. In making this calculation, Manning's  $n$  of 0.01 was assumed to calculate  $S$ . Based on this order of magnitude for the film thickness, it is seen that some of the data was probably taken near or within the film. Even though it

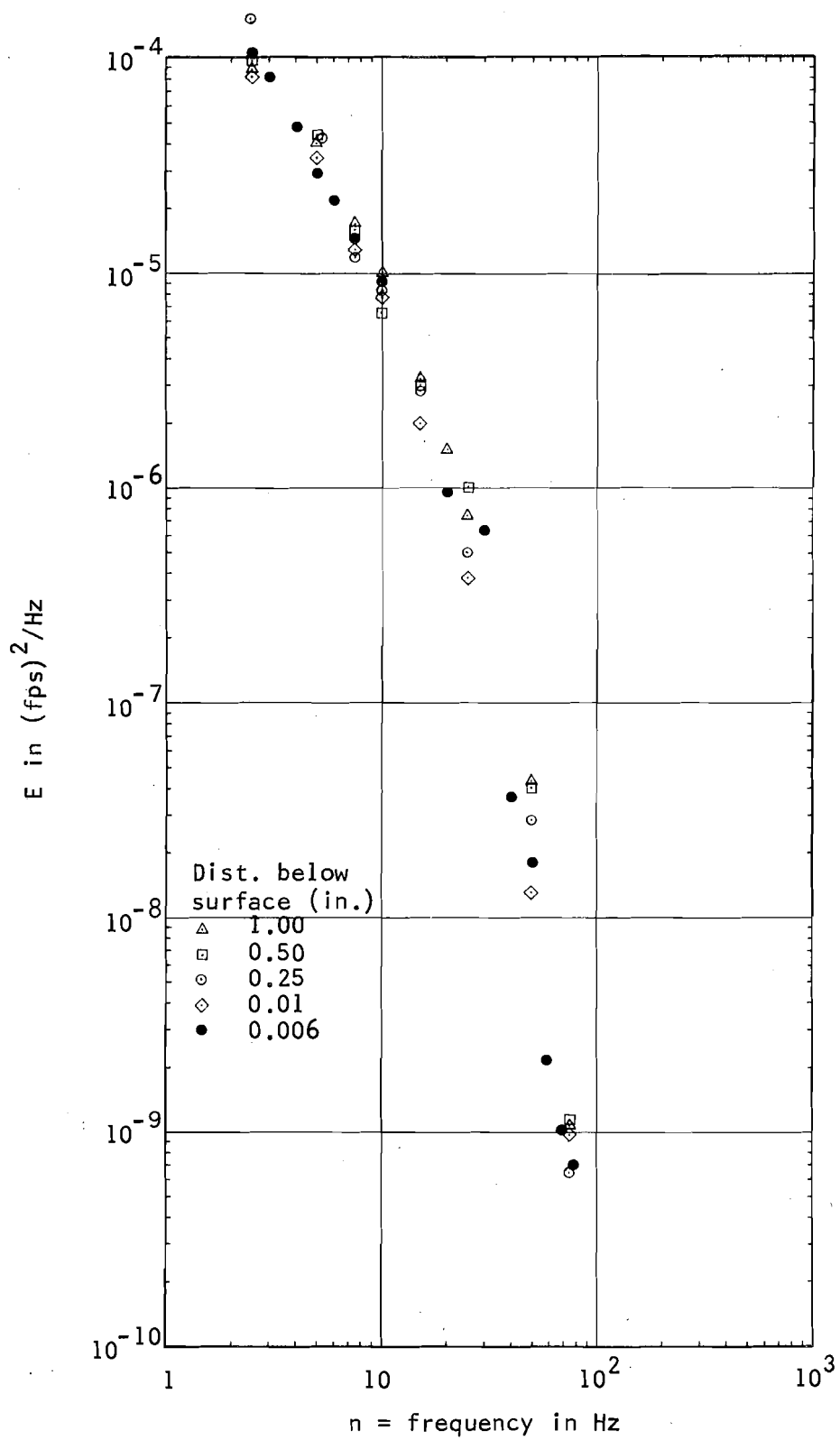


Fig. 4-5: Energy Spectra

is not possible to take data right at the surface, it seems unlikely that the character of the turbulence would undergo any gross change in a distance of 0.01 in., even immediately below the free surface. The presence of small waves on the surface seems in itself to be a testimony to the fact that turbulence exists at the free surface (26).

At a frequency of 2.5 Hz, the spectral distribution has a slope which indicates the presence of significant energy in lower frequencies (Fig. 4-5). With the instruments used in these tests, it was not possible to measure the lower frequency components.

The RMS of the velocity fluctuations was measured as approximately 0.021 fps by Circuit B, Fig. 4-3, for all positions in the upper inch of the water. This corresponds to an intensity of 0.020 based on the local mean velocity of 1.08 fps. As a result of the exclusion of the low frequency components from the analysis, the measured values of the RMS were no doubt too low. For example, Raichlen (21) measured an intensity of approximately 0.04 at a relative depth of 0.8 (0.77 in. below the free surface) for a Reynolds number of 91,100.

The actual RMS value in the upper inch for the present tests was estimated to be 0.035 fps. This value was obtained by extrapolating the spectra (E) to  $n = 0$  by a curve geometrically similar to that for the spectra obtained by Raichlen and then integrating the area under the curve to get the RMS fluctuation. This is a rather inaccurate procedure, at best, since about 1/3 to 1/2 of the area lies under the extrapolated part of the curve. This also indicates the need in future tests for making provision for including frequencies less than 2.5 Hz in the measurements.



An attempt was made to extrapolate the spectral distribution by a least-squares fit of the data to Eq. 4-4 to find values of  $L_x$  and  $E(0)$ . This approach led to negative values of the integral scale  $L_x$ , apparently indicating that too few data points were available and/or the scatter in the points was too great and/or Eq. 4-4 does not describe the functional variation of the data. This latter reason seems plausible in view of the comparison between Raichlen's data (21) and Eq. 4-4.

Using the estimated RMS (0.035 fps) and the local mean velocity of 1.08 fps, the spectral data is shown normalized in Fig. 4-6. Data from Raichlen for open channel flow (21) and Laufer (15) for two-dimensional closed conduit flow are also shown. In view of the gross approximations made in the analysis of the data, the agreement is somewhat surprising.

The purpose of making these turbulence measurements was to investigate any possible change in the turbulence structure in the vicinity of the free surface in view of the steep concentration gradients which apparently exist in the so-called "film". The data in Fig. 4-5 tend to indicate that there is no significant change in the turbulence structure in the region immediately below the free surface. Thus, apparently, the steep gradients and the low diffusion rates in the film region (Section 3) are not caused by a decrease in the turbulence level at the free surface. Even though data was not taken on the portion of the spectrum at frequencies less than 2.5 Hz, rapid vertical changes in these components would certainly not be expected since the low frequencies roughly correspond to the large eddies.

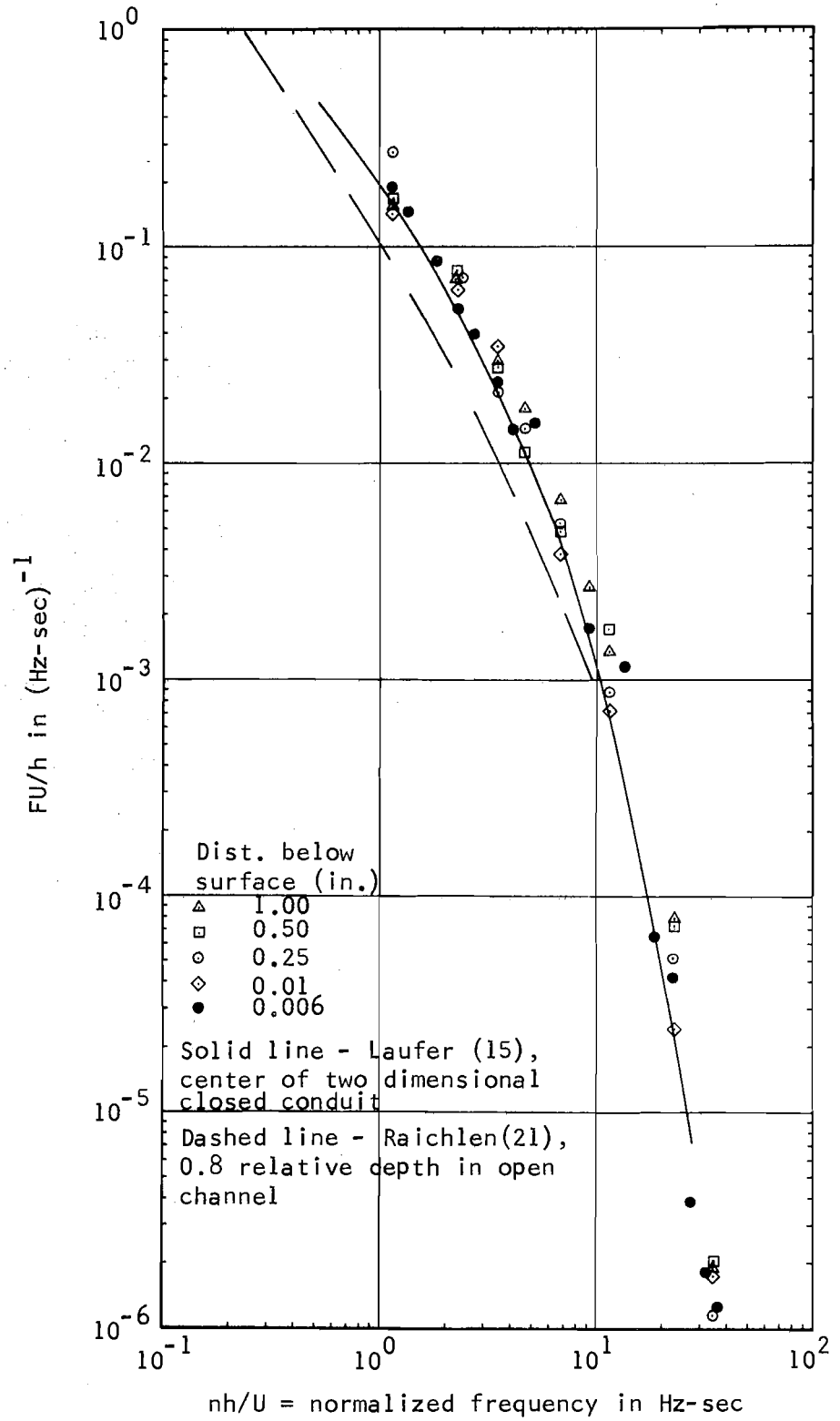


Fig. 4-6: Normalized Energy Spectra

## 5) CONCLUSIONS

Reaeration rate coefficients have been related to flow parameters primarily by empirical and semi-empirical approaches in the past. As yet, these approaches do not seem to have produced relationships which enjoy general acceptance or which are able to correlate consistently with both laboratory and field data. Thus, there seems to be a need to investigate the basic mechanisms which influence the reaeration rate.

If the water surface is at saturation concentration, the rate of reaeration is controlled by the rate of downward transport of oxygen away from the free surface. Measurements, using heat (temperature) as a tracer, indicate that it may be possible to represent the downward transport by a diffusion model. With this model, the rate of reaeration is completely defined if the diffusion coefficient and the steepness of the concentration gradient are both known at the water surface. Limited measurements in a mixing vessel with a heat tracer indicate that the diffusion coefficient at the surface may be equal to or greater than the molecular diffusion coefficient, but that it is considerably less than would be expected from the amount of mixing present at the free surface. The measurements indicate a linear temperature (concentration) gradient immediately below the free surface.

Since transport at the free surface is apparently the critical mechanism in the reaeration process, it would seem to be beneficial to investigate the flow conditions immediately below the free surface. Turbulence measurements were made from 0.006 in. to 1 in. below the free

surface in a laboratory flume with 6 in. flow depth and 1 fps velocity. The turbulence spectra indicated no change in turbulence structure in this region. Thus, apparently, turbulence exists right up to the surface and in the region often referred to as the surface "film".

Much more work needs to be done on the mechanisms which control reaeration rates. Only a very limited amount of reliable data was obtained during this study.

Thus, the conclusions presented above are somewhat tentative. More work is needed to investigate the validity and usefulness of the diffusion model and to seek relationships of both the diffusion coefficient and the concentration gradient to the turbulence conditions and ultimately to mean flow parameters. It would probably be instructive to make simultaneous measurements of turbulence and diffusion near the free surface of open channel flow. This should allow a better correlation of the diffusion coefficient to turbulence conditions and allow comparison of parameters such as the apparent film thickness and the microscale of the turbulence.

Acknowledgements. The experimental part of this work was carried out by Saeed Massoudi, an undergraduate student, and A. Keramati, a graduate student.

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## SYMBOLS

$A_s$	surface area
$a$	amplitude of screen motion
$b$	thickness of surface film
$C$	average concentration in bulk of water
$c$	concentration at a point
$c_s$	saturation concentration
$D$	oxygen deficit = $c_s - C$
$D_n$	molecular (mass) diffusion coefficient
$E$	one-dimensional energy spectrum
$e$	diffusion coefficient
$F$	normalized one-dimensional energy spectrum
$h$	water depth
$K_1$	deoxygenation rate coefficient
$K_2$	reaeration rate coefficient (per unit volume)
$k_2$	$K_2/2.30$
$K_L$	reaeration rate coefficient (per unit surface area)
$k_L$	$K_L/2.30$
$L_x$	macro length scale
$n$	frequency
$q$	transport rate
$R$	resistance
$r$	surface renewal rate
$S$	friction slope
$t$	time, temperature

$U$	average velocity
$u$	x-component of velocity
$\bar{u}$	mean value of $u$
$u'$	turbulent fluctuation = $u - \bar{u}$
$V$	volume
$x$	longitudinal coordinate
$y$	vertical coordinate
$\rho$	temperature coefficient
$\sigma$	frequency of screen motion
—	time average